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Journal
of
The Franklin Institute
Devoted to Science and the Mechanic Arts

Vol. 242 AUGUST, 1946 No. 2

SILICONES: FOOD FOR IMAGINATION.

BY

R. R. McGREGOR,

Senior Fellow, Mellon Institute, Pittsburgh, Pa.

JOINT MEETING OF THE FRANKLIN INSTITUTE AND THE PHILADELPHIA SCIENCE TEACHERS ASSOCIATION—MAY 1, 1946.

The opportunity to address a group such as this is not a common one. The very name of the Franklin Institute brings to mind the long and illustrious history of scientific development in America. To be invited to speak in its halls is at once an honor and a responsibility. When to this is added the invitation from the Science Teachers Association of Philadelphia, it is necessary that one should be prepared to offer information and, if possible, mental stimulation. I have accepted your invitation in the hope that I could in some fashion at least measure up to the opportunity and the responsibility.

I realize that the group here is not composed entirely of chemists, nor entirely of engineers nor physicists nor astronomers, but a cross-section of those who are interested in advances in scientific work and technology. For this reason I do not propose to dwell too long on the chemistry of these materials, for that might not interest everybody. I will try to illustrate my points by showing you the products developed, and let you ask yourselves how they came to be or what shall we do with them. This may be of particular interest to teachers of science, for by presenting the new or the unusual to the student it is possible to arouse his curiosity and stir his imagination. When curiosity is aroused and imagination stirred there comes a *demand* rather than a mere *desire* for information and understanding, and the development of the student is limited then only by his mental capacity. .

THE CARBON-SILICON PROGENY.

The field of chemistry is generally considered as divisible into two sections: organic, or the chemistry of carbon compounds; and inorganic, which covers the chemistry of all the rest of the elements. When

(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the JOURNAL.)

thought about in this way it appears that carbon considers itself rather exclusive and a law to itself.

Carbon chemistry not only appears to be exclusive, but in some ways seems to think of itself as the aristocrat of the elements. It has been systematized, organized and advertized. The chemistry of silicon has not had the notoriety of carbon, and this element may be considered a modest and unassuming member of the proletariat. It is a product of the soil, found in sand, bricks and mortar—a very humble member of society. In human relations intermarriage between different ethnologic groups is generally frowned upon, for you never know what will come of it. This may be the reason that the families of carbon and silicon were kept apart for so long. But the union finally took place, and as a result we find a very unusual and happy progeny. At times some of them act like one of the parents, and at times like the other. On still other occasions they take on characteristics of their own. There is a complete analogy here to human behavior. Many of the children show no special aptitudes, some are brilliant in special lines, and we have at least one playboy. I would like to tell you something about this family, pointing with pride to the more promising and raising an eyebrow at the eccentric. I would like to invite you to use your imagination in a number of directions. Those of you who care to consider the pertinent chemistry may inquire into the basic reasons for some of the peculiar characteristics and actions of these offspring (for the Emily Posts of the carbon and silicon families would be shocked at some of the actions of their children); those of you who care to consider the physical differences may inquire into the reasons for unorthodox physical properties; those of you who are more interested in effects than causes may care to let your imagination take wings and, accepting the fact that many of the members of this family refuse to follow the rules that governed their parents, think of what can be done with them to make them useful members of society.

CHEMICAL GENEALOGISTS.

To continue the analogy we may speak of those who have studied these compounds as chemical genealogists. For a long time they spent their energies trying to show that these children really resembled their parents. And, of course, when we have our family tree investigated we always like to have it demonstrated that we are direct descendants of an emperor or a duke or some type of aristocracy. Hence for years these chemical genealogists tried to show that these organo-silicon compounds resembled aristocratic carbon more closely than plebeian silicon. Foremost among these investigators was Professor F. S. Kipping of the University of Nottingham in England. He wanted to demonstrate that silicon could be optically active, just as carbon can be. He succeeded, but it was a struggle. Over a period of 40 years, from 1900 to 1940, he

investigated this family—and he found it exasperating. Some of the members acted like the carbon family, some acted like the silicon family, some acted like no known family at all—and we can imagine that he was tempted to throw them in the sink in disgust. However, he published his results—51 papers in all—thus introducing them to the world and letting those who would cultivate them or ignore them.

A NON-CONFORMING FAMILY.

Within the past ten years or so their acquaintance has been cultivated by several groups of chemists. But at the beginning of what we may call this new social life they entered, they were still expected to behave like their parents. It was thought, as they showed such a high order of resistance to oxidation and could be induced to assume sticky resinous forms, that perhaps they could be transformed into a very special type of resin—a more resinous resin—an extrapolation to the nth degree of the properties of resins as we knew them. Or into a more carbonaceous carbon—or a more siliceous silica—perhaps actuated by the same general philosophy which Rupert Brooke's fish enunciated as he pondered the problem of a future life; for he too was not able to picture anything in the future which he had not experienced to some degree already:

> Fish say, they have their Stream and Pond;
> But is there anything beyond?
> This life cannot be All, they swear,
> For how unpleasant, if it were!
> One may not doubt that, somehow, Good
> Shall come of Water and of Mud;
> And, sure, the reverent eye must see
> A purpose in Liquidity.
> We darkly know, by Faith we cry
> The future is not Wholly Dry . . .
> But somewhere, beyond Space and Time
> Is wetter water, slimier slime.

But events showed that we had come upon a group that refused to be governed by the old mores—that conformed when it chose, and deviated when it chose—a conservative or a left-winger.

ORGANO-SILICON SYNTHESIS.

Perhaps as good a way as any to introduce you to this family would be to show how these compounds are produced. Many of you are familiar with the processes and I will ask your indulgence while I try to give a brief explanation so that we will all be on the same ground. I realize that everyone here is not a chemist, but I believe that the method of synthesis can be described very simply. There are a number of methods that can be used, but the classical and also one commercial method of preparing them is by the use of the method developed by

Grignard and known as the Grignard reaction. In this method magnesium chips are covered with dry ether. An organic halide such as phenyl chloride is dripped into this while it is stirred. The temperature rises, the ether refluxes, and in time the magnesium disappears. Under the best conditions a clear solution results. The reaction is described as:

$$RCl + Mg \rightarrow RMgCl.$$

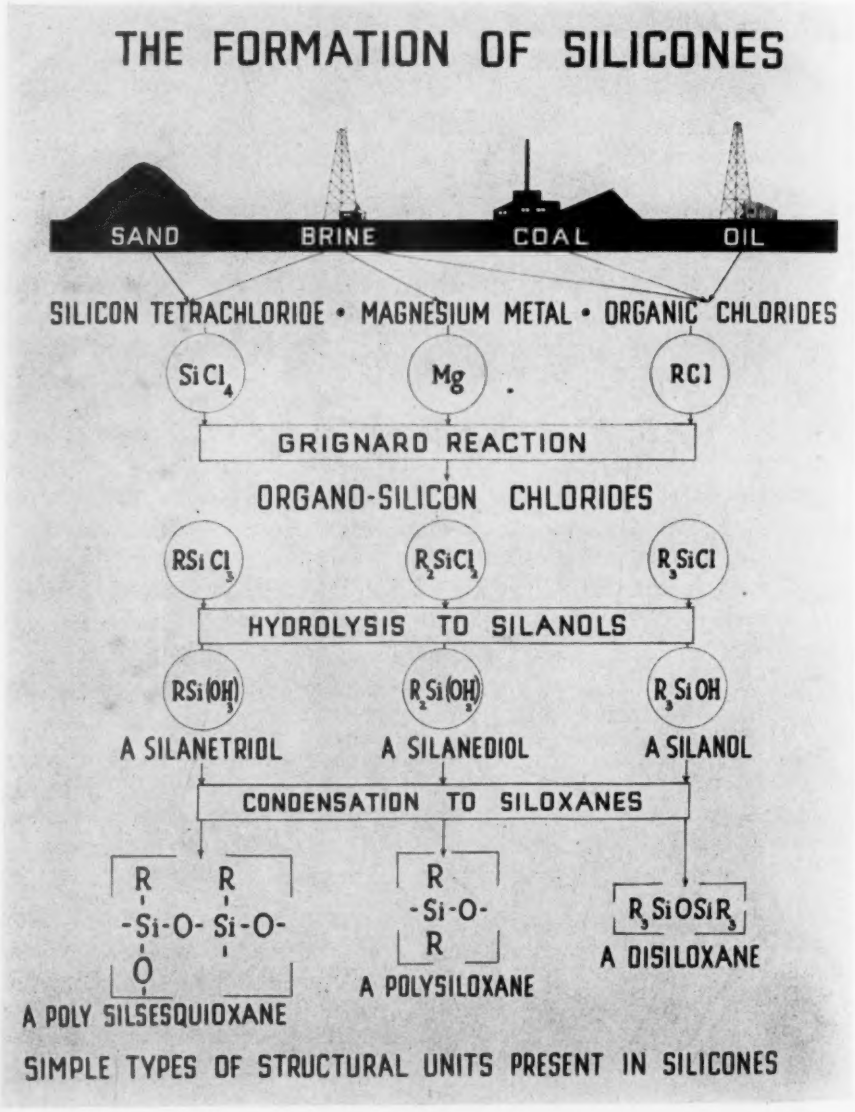


FIG. 1. Diagram of processes and intermediaries involved in the formation of silicones.

If then this solution is dripped into silicon tetrachloride in ether solution, magnesium chloride separates out as a result of the reaction:

$$SiCl_4 + 2RMgCl \rightarrow R_2SiCl_2 + 2MgCl_2.$$

The R_2SiCl_2 may be distilled off and is the raw material for preparation of a silicone.

If this is treated with water there is an immediate hydrolysis of the chlorines giving:

$$R_2SiCl_2 + 2H_2O \rightarrow R_2Si(OH)_2 + 2HCl.$$

If that were all there was to it Kipping would have had a good time, for he could have isolated the $R_2Si(OH)_2$ or any other similar product he cared to make, determined the properties, and showed its position in the family tree.

But these compounds are not so obliging. The $(OH)_2$ groups are unstable and split out water, leaving an oxygen. Even this wouldn't be too bad for then we would have R_2SiO or a ketone type. It seems to be a strange thing, but silicon never is combined with oxygen by a double bond except perhaps in the vapor state. So what we really have is O-SiR—O-SiR, which is a polymer. This was Kipping's greatest worry and the industrialist's greatest hope.

It is the control of this polymerization that spells success or disaster for the industrialist; for these various polymers carried to just the proper polymer size are the articles of commerce.

PROPERTIES OF FLUID POLYMERS.

And now to look at some of the progeny of these fruitful parents.

We may consider first the group of fluid polymers. Here we have a liquid that looks like highly refined petroleum oil. We should expect it to resemble the carbon parent; but if it is analyzed it will show about 80 per cent silica. If it contains that much silica it should at least solidify by being chilled; but it doesn't freeze until it reaches 40 degrees below zero; and here is another that doesn't freeze until it reaches 100 degrees below zero F. Why is this? Both the carbon compounds and the silicon compounds generally freeze far above these temperatures. We are beginning to get hazy ideas as to the reasons back of this, but someone with insight and with imagination must yet supply the complete answer.

Not only do these liquids have low freezing points, but they show less change of viscosity with temperature than any other liquid.

They resist oxidation and have had pure oxygen bubbled through them while they were held at 200° C. After hours of this drastic treatment the change in properties was scarcely measureable.

When I add to these statements that the electrical properties are unusually good, you may have some food for imagination as to what is the cause of all this and what you can do with such materials.

A few demonstrations of the properties of these fluids and allied materials will be of interest and may stir your imagination more than anything I can say. (At this point there was demonstrated the water-repellent properties of these materials on paper, glass and porous brick; also the antifoam properties in oils and in soap solutions.)

So much for the fluid polymers.

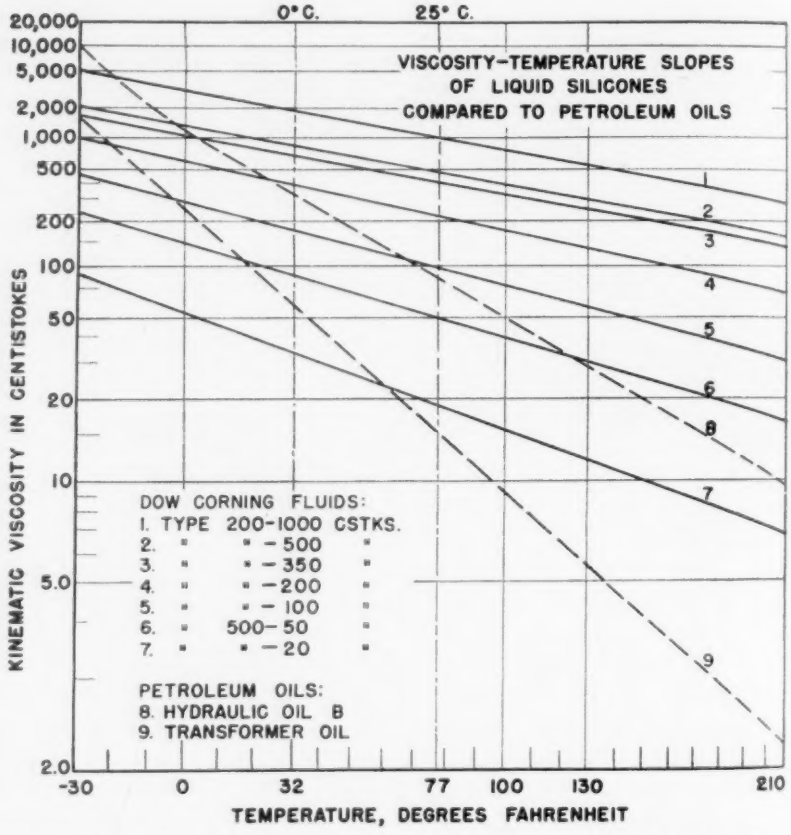


FIG. 2. Graph showing comparison of viscosity-temperature slopes of silicone fluids and petroleum oils.

ORGANO-SILICON GREASES.

It is possible to compound these fluids into greases. Many of the properties inherent in the fluids are still present, such as water repellency, oxidation resistance, etc.; but some of them are enhanced—for instance, the change of viscosity with temperature. Within some kind of reasonable limits—40 below to 400 above—there is no change in viscosity at all. They don't run with heat, although it is possible, at quite elevated temperature, to make them burn. Their lack of flow coupled with good electrical properties makes them good dielectrics for

hot locations. They stay put! They are useful as lubricants in places where heat would smoke off ordinary petroleum lubricants. Why don't they flow with heat? There may be food for imagination here in supplying the answer, and similarly there may be food for imagination in thinking what can be done with such materials.

HEAT RESISTANT RESINS.

The greases show no flow with temperature, but they are soft.

Here is another type of polymer that shows no flow with temperature, but it is solid. This is a piece of Fiberglas cloth impregnated with a silicone resin. It is flexible, heat resistant, water repellent, and an excellent electrical insulator. It surprises a great many people to learn that the only function of an electrical insulator is to keep out water. Well, then, what's the matter with rubber? Not a thing, as long as it stays rubber. But a motor or any piece of electrical apparatus that is worked hard or is subject to overload becomes hot. If it becomes very hot the rubber softens, oxidizes, decomposes, lets water in, there is a "short," and the motor burns out. The advantage of the silicone insulation is that it does not decompose except at quite high temperature; therefore it continues to keep water out even though the equipment is heavily overloaded. Motor tests are in progress right now where the temperature is held above 300° C. It is frequently shut down and held at 100% humidity until the motor is dripping wet. The paint has been burned off the frame; but the motors are still functioning smoothly for the insulation is remaining intact and is keeping out the water.

Similar resins can be used to bond layers of glass fabric together to provide panel boards for electrical instruments. If an arc is formed as a result of high voltage no carbon track is formed, but rather a silica track which gives even better insulation.

May I raise another question right here for your imagination to work on? When the general run of carbon containing compounds are raised to 250° or 300° C., they oxidize fairly readily and CO_2 is formed. But if the carbon is attached to silicon it is much more difficult to bring about oxidation. If one carbon atom (methyl) is attached, oxidation is very difficult; but if two carbons in a chain (ethyl) are attached, oxidation is easier. As this chain is lengthened, oxidation progresses more and more readily. Why does silicon exert such a protective action within its sphere of influence?

This oxidation protective action is shown where the resins are formulated into paints. Such paints remain coherent and protective long after the best of the organic paints have flaked off and disappeared. One nice place to use the new paint in the home is on the hot water heater. After a test of a year in one case with which I am acquainted the heater looks as bright and shiny as the day it was applied. Why

they are so heat resistant and what should be done with them I leave to your imagination.

THE SKELETON IN THE CLOSET.

But every family has some sort of a skeleton in the closet—perhaps a grandfather who was addicted to strong drink, or an uncle who ran off

FIG. 3. The badly rusted exhaust muffler to the right was painted with conventional aluminum paint on the same day that the muffler next to it was coated with Dow Corning Silicone Resin pigmented with aluminum powder. Both mufflers have been exposed for over a year to surface temperatures in the range of 430° to 510° F.

with an actress—or a cousin who voted the Democratic ticket. There's a certain lurid fascination about them, but they really reflect very little credit, and they are rarely spoken of outside the family circle.

But it seems only fair tonight, since we are looking at the carbon-silicon family with an appraising glance, that we should take note of at least one of the skeletons in the closet. I refer to the silicone man-about-town, commonly referred to as Bouncing Putty. He seems quite irresponsible, breaks the laws and does no useful work. He is tolerated because he is an eccentric. As with most of such folks he is recognized as a member of the family only on social occasions. Some day he may settle down and be of some use in the community, but so far he seems scarcely worth the trouble it took to raise him.

This Bouncing Putty can really be looked at in a more serious vein. I said before that he breaks all the laws—and that is just about what I meant. It doesn't correspond with our experience to find a material that can be drawn out into long threads and has cold flow, that can at the same time be readily fractured or broken. And then to find that this material has a high rebound when dropped—well, it just doesn't add up. It is an interesting fact that children see nothing unusual about it. It's fun, because they can do different things with it. But they find nothing paradoxical about it. To those who have had some experience in the testing and correlation of physical properties it presents some interesting problems. Your imagination might be stirred by consideration of such a material as this. Some of you may inquire what are the reasons behind these peculiarities; others, accepting the fact that these properties are present, may want to ask what you can do with them. In spite of this peculiar combination of properties *we* don't know what to do with it.

RUBBER WITH 90 PER CENT ASH.

There is one more member of this family I would like to introduce to you. It is known as Silicone Rubber or Silastic. You will note that it looks like a white rubber. It stretches, it bounces, it makes a good waterproof. It must be rubber! But if you burn this down to an ash you will find that you have left not the 5 per cent to 10 per cent usually left by rubber, but about 90 per cent. In other words, it is nearly all inorganic. What manner of thing is this, that has so many of the physical properties of an organic material and yet declares itself on analysis to be nearly all inorganic?

It is thought of as a rubber because it stretches and bounces, but there the analogy ceases. It does not soften on heating as rubber does; it does not freeze as readily upon drastic cooling; it is not affected by petroleum oils. It is very resistant to oxidation and ozone. Chemically it does not even remotely resemble rubber, for it has no double bonds and is not vulcanized by sulfur. Rubber has been studied carefully

over many years and so far there is no theory about its structure that is universally accepted. The Silicone Rubber, or Silastic, is not any more willing to yield its secret. There is plenty of room for imagination here.

ANOTHER FRONTIER OF KNOWLEDGE.

The materials described and shown comprise the main types of compounds so far prepared. If time permitted the list could be extended.

It has not been my purpose tonight to tell you all about these materials, but rather to present them in such a way that you may see the peculiar position they occupy as chemical compounds and as engineering materials. To tell you the truth, we're a bit puzzled ourselves about what some of them are, why they behave as they do, and what to do with them. I may as well admit that it's fun to work with them. They provide plenty of room for chemical investigation and for engineering imagination, for you can take nothing for granted.

Addressing myself more particularly to teachers of science, I could suggest that the description of such materials to the student might help him to realize that the last frontiers of knowledge and invention have not been passed—that more and more lies beyond the horizon. Other problems and products not thought of now can without doubt be found, and the excitement of search and discovery awaits those with energy and active imagination.

STUDIES IN THE SENSITIVITY OF PHOTOGRAPHIC MATERIALS. V. THE EFFECTIVE GRAIN SURFACE OF LATENT-IMAGE FORMATION.

BY

A. P. H. TRIVELLI,

Communication No. 1084 from the Kodak Research Laboratories.

In Part III [1] of this series which dealt with the intrinsic sensitivity of photographic materials, investigations of the effect of exposure to X-rays upon the developability of grains of a photographic emulsion were described. The experimental emulsion used contained silver bromide grains which, with the exception of the smallest, consisted of flat tablets all of the same thickness. One-grain-layer coatings of this emulsion were exposed in a series of sensitometric steps. Development was carried out in a special hydroquinone developer which produced pseudomorphic grains, i.e., developed grains having the same size and shape as the undeveloped grains, indicating a surface reduction of the affected grains; no fixing bath was employed so that the undeveloped grains were not removed.

Photomicrographs of the most suitable fields were chosen for preliminary inspection to show which grains had become developable by exposure. One of these pictures, which was made originally at a magnification of 2,500 times, is reproduced as Fig. 1.

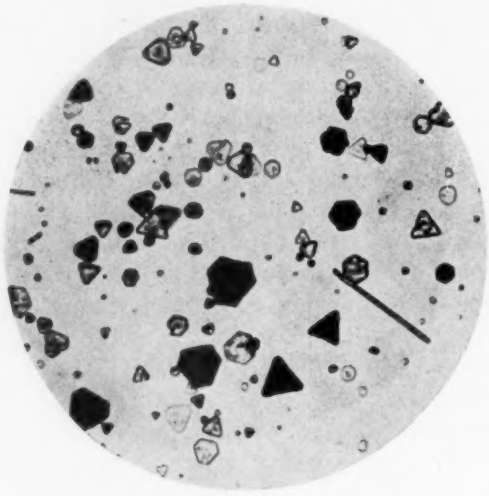
All exposures were made with radiation uniformly distributed over the whole surface of the exposed areas. The energy absorbed by the different grains during exposure is, therefore, directly proportional to their projective areas. It is, however, easy to find many cases in Fig. 1 in which small grains have become developable while larger grains remained undeveloped, and vice versa. Does this mean that these small grains are more sensitive than many of the larger grains? According to the definition that the sensitivity of a grain is measured by the reciprocal value of the total energy of the incident radiation required to make it developable, the answer could only be affirmative. However, such comparisons between a small number of grains do not lead to any such definite conclusion. The picture becomes quite different if the grains are classified according to their sizes and the percentage of developed grains of each size is determined. Then it is immediately evident that after a certain exposure time a greater percentage of larger grains has become developable.

In other words, the determination of the relation between grain

[1] A. P. H. Trivelli, *J. Franklin Inst.*, **241**, 85 (1946).

sensitivity and grain size requires a statistical method of investigation. The definition given of the sensitivity of a grain must, therefore, be changed accordingly. Statistically, it is measured by the probability that a grain will become developable at a certain exposure.

The statistical investigation of grain sensitivity was introduced by Svedberg [2] and by Silberstein [3] independently of each other. Svedberg's method of investigation was experimental; Silberstein's method was theoretical, in which he gave physical meaning to all parameters in his equations. Renwick [4] questioned the validity of the statistical



2500 ×

FIG. 1. Pseudomorphic silver grains obtained by special development of grains affected by an X-ray exposure.

method of investigating grain sensitivity. His experience in emulsion-making had convinced him that a direct relation did not exist between grain size and sensitivity. He gave as evidence photomicrographs of grains of two emulsions "produced under apparently identical conditions," having quite different grain shapes and grain sizes. The small-grain emulsion had nine times the inertia speed of the large-grain emulsion. Slater-Price [5] questioned the identity of the preparation of the two emulsions and so does the present author. It has never been revealed how Renwick produced this experimental effect, which is contrary to the present author's experience in photographic emulsion re-

[2] (a) T. Svedberg, *Z. wiss. Phot.*, **20**, 36 (1920); (b) T. Svedberg and H. Anderson, *Phot. J.*, **61**, 325 (1921).

[3] L. Silberstein, *Phil. Mag.*, **44**, 257 (1922).

[4] F. F. Renwick, *Phot. J.*, **61**, 333 (1921).

[5] T. Slater-Price, *Phot. J.*, **61**, 335 (1921).

search. Renwick never gave any information on how he made his emulsions. He offered only photomicrographs of the grains. It is, therefore, practically impossible to confirm his experimental results.

Renwick's statement, however, emphasized the important fact, brought out by his remarks and many times confirmed by the experiences of emulsion-makers, that the simple relations between grain size and grain sensitivity among grains in a single photographic emulsion do not hold between grains of different emulsions. However, Jensen and the author [6] were able to show that a statistical relation of a complicated nature exists between the average grain size and the inertia speed of fourteen commercial emulsions chosen at random. They determined the correlation coefficient (k) and its probable error (Q), and found that

$$k = 0.51 \pm 0.013 \text{ and } k/Q = 3.82,$$

while the criterion (P) of the probability that this result is mere chance was calculated and found to be

$$P = 0.01006.$$

In general, therefore, high-speed emulsions have larger average grain sizes than low-speed emulsions.

These investigations were made with old-type emulsions. During the last ten years new types of emulsions having higher inertia speeds and smaller average grain sizes have appeared. The result just mentioned holds true only for the old-type emulsions. The present author's experience with emulsions, however, leads him to believe that a similar conclusion holds true for the relation between inertia speed and average grain size among new-type emulsions.

Svedberg [2a] measured the grain sensitivity by counting the average number of development centers per square unit of the projective area of the grains resulting from a controlled exposure. These centers were made microscopically visible by the initial development of exposed grains. The fact that the number of developed grains increases on prolonged development indicates that only a small fraction of all developable grains will develop at the initial stages of development. The value of Svedberg's method of measuring grain sensitivity is, therefore, seriously limited.

There is also another objection to this method of measuring grain sensitivity. The formation of a single development center by exposure is sufficient to cause development of the whole grain. The presence of additional development centers per grain does not change the final result. This was realized by Silberstein [3] in his statistical definition of grain sensitivity. Silberstein's treatment offers a method for investigating theoretically how many of the developable grains will de-

[6] A. P. H. Trivelli and E. C. Jensen, *J. Franklin Inst.*, **209**, 37 (1930).

velop at different stages of the development process until maximum gamma is reached. At each development time the grain sensitivity per square unit of the projective area of the grain in an assemblage of grains can be determined quantitatively. Hence the validity of Silberstein's method of measuring grain sensitivity is greater than Svedberg's.

Toy [7] defined the grain sensitivity as the sensitivity of its most sensitive sensitivity speck. The formation of specks on the grains of photographic emulsions by the reaction of a sensitizer in aqueous solution is to be expected. All chemical reactions between solids and liquids, and between solids and gases start with the formation of nuclei on the solids. Analogous reactions between dehydrations, decompositions, and even explosions of solids also start from nuclei. Sheppard, Trivelli, and Wightman [8] obtained photomicrographic evidence of speck formations on silver bromide grains in photographic emulsions by using a properly diluted thiocarbamide solution as sensitizer. The presence of gelatin did not prevent speck formation.

The agreement between the spectral sensitivity distributions of photographic emulsions and the absorption spectra of the silver halides, as shown by Huse and Meulendyke,[9] and Toy and Edgerton,[10] points to the fact that the sensitivity speck of silver sulfide is not the sensitive material in latent-image formation.

In Part III [1] of this series, which offered a tentative explanation of the intrinsic sensitivity of photographic materials in general and of the grain in particular, the fact was recalled that by the exposure of silver halide grains only a very small measurable fraction of the incident radiation, ϵ, is used for latent-image formation. This indicates that only a very small fraction of the projective area ($a\mu^2$) of the grain, $\epsilon a\mu^2$, is used to initiate developability. The evidence just given leads inevitably to the conclusion that the effective grain surface, $\epsilon a\mu^2$, in latent-image formation cannot be the projective area of the sensitivity speck.

Accepting the Gurney-Mott hypothesis [11] of the atomic mechanism of latent-image formation as offering the most promising explanation, to date, of the concentration-speck hypothesis of Sheppard, Trivelli, and Loveland,[12] we can now extend the tentative interpretation of the relation between the effective grain surface and the grain sensitivity, which was started in Part IV [13] of this series. From this discussion it is clear that the most probable picture of the effective grain surface consists of a silver sulfide speck surrounded by a potentially effective

[7] F. C. Toy, *Phil. Mag.*, **45**, 715 (1923).
[8] S. E. Sheppard, A. P. H. Trivelli, and E. P. Wightman, *Phot. J.*, **67**, 281 (1927).
[9] E. Huse and C. E. Meulendyke, *Phot. J.*, **66**, 306 (1926).
[10] F. C. Toy and H. A. Edgerton, *Phil. Mag.*, **48**, 947 (1924).
[11] R. W. Gurney and N. F. Mott, *Proc. Roy. Soc.*, **164A**, 151 (1938).
[12] S. E. Sheppard, A. P. H. Trivelli, and R. P. Loveland, *J. Franklin Inst.*, **198**, 51 (1925).
[13] A. P. H. Trivelli, *J. Franklin Inst.*, **241**, 315 (1946).

silver halide surface from which the speck collects the disintegration products by exposure to make a development center from the sensitivity speck.

The action of a sensitizing gelatin upon silver halide produces specks of different sizes on the grain surface. This is comparable to the grain formation resulting from the precipitation of silver halide in a gelatin-alkali-halide aqueous solution by a silver nitrate solution in water. The grains are formed on nuclei which are distributed at random in the gelatin, alkali halide, and silver nitrate solutions.

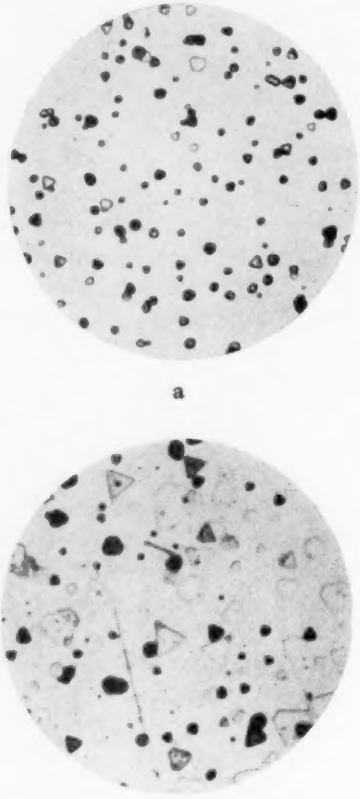
It is generally assumed that the topochemical effects on crystals start at the points of discontinuity in the crystal structure, of which there are many in each crystal. These places are also distributed according to the laws of probability favoring the crystal corners and edges. Since in photographic emulsions, the growth of grains is proportional to the surface of the grain, a similar growth of the sensitivity specks is to be expected during sensitizing of the emulsion, resulting in skew size-frequency curves.

All these sensitivity specks are surrounded by surfaces that are potentially effective in latent-image formation. The larger specks may be assumed to be surrounded by larger potentially effective surfaces. The larger grains having statistically larger values of ϵ will also have statistically larger silver sulfide specks. We have, then, a number of potentially effective surfaces upon a sensitized grain which are distributed according to skew size-frequency curves having the same shapes as those of the grain-size distribution curves of silver halide emulsions.

If the grains of a photographic emulsion are spread at random over the surface of a microscope slide, we obtain a grain-size distribution of the silver halide crystals controlled by the same probability law as the distribution of different sensitivity specks on a grain surface. The photomicrographs of Figs. 1, 2a, and 2b may serve as examples of such probable distributions around the centers of the grain surfaces. If each grain in these pictures is replaced by a speck having the same projective area as the grain, we get a many times enlarged image of the probable distribution of sensitivity specks. We do not know, however, how many sensitivity specks there are on each sensitized grain. Grains of the same size, taken in sufficiently large numbers, however, will give smooth speck-size distribution curves. Just as it is not necessary, but only probable, that by exposure larger grains become developable before smaller ones, so larger sensitivity specks have a better chance of becoming development centers than smaller ones. The location of an effective surface depends on where the required number of effective quanta are absorbed.

In connection with these hypothetical conclusions, W. F. Smith and the author made a few experimental investigations. Two emulsions were prepared under conditions as nearly identical as possible, with the

exception of the silver iodide content. This resulted in a difference between the skew grain-size distributions of the two emulsions. The total surface of the grains per cubic centimeter was 1,015 square centimeters for the first emulsion and 1,472 square centimeters for the second. Half of each emulsion was left unsensitized and the other half was sensitized with a gelatin containing a sulfur sensitizer. In both cases,



a

b

2500 ×

FIGS. 2a and 2b. Silver halide grains haphazardly distributed over the surface of an object glass.

the sensitizing conditions were the same since the optimum effect was obtained by using the same amount of sensitizing gelatin. All four emulsions were coated to the same thickness on a film base.

Each of the films received the same sensitometric exposure. Development was accomplished in all cases at 18° C. for 12 minutes in DK-50 developer. With this development time virtually maximum gamma was reached. The fog-corrected characteristic curves obtained

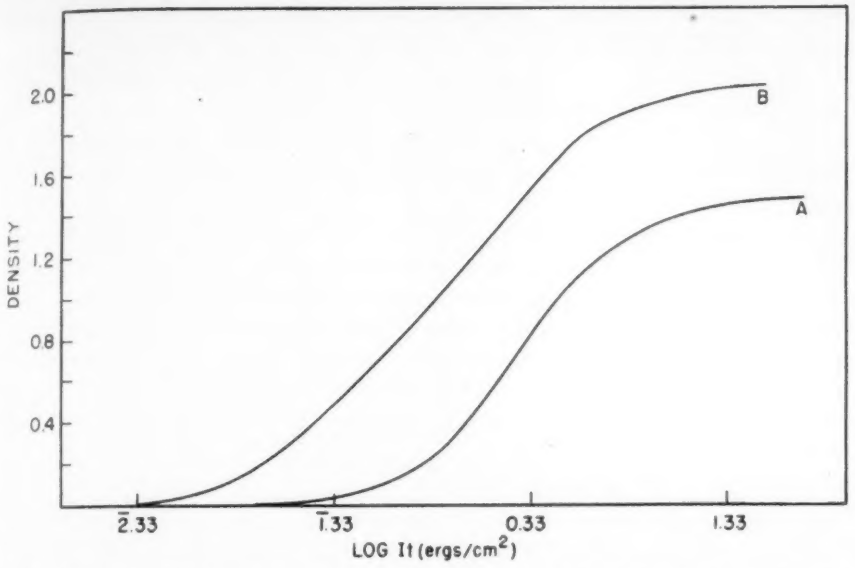


FIG. 3. Characteristic curves of emulsions with a total grain surface of 1015 cm² per cubic centimeter of emulsion: (A) = unsensitized. (B) = sensitized.

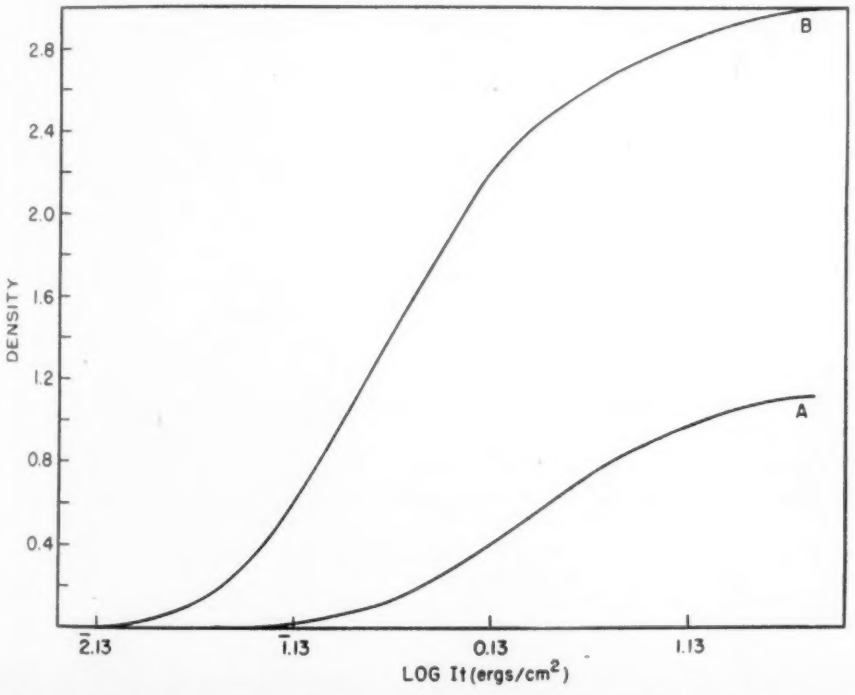


FIG. 4. Characteristic curves of emulsions with a total grain surface of 1472 cm² per cubic centimeter of emulsion: (A) = unsensitized. (B) = sensitized.

are given in Figs. 3 and 4. Curves *A* were produced by the unsensitized emulsion and Curves *B* by the sensitized emulsion. Curve *A*, Fig. 3, having the smaller total grain surface, is a simple two-quantic curve. Curve *B* for the sensitized emulsion is a two-two-quantic curve. Curve *A*, Fig. 4, having the greater total surface, is a simple one-quantic curve. Curve *B* for the sensitized emulsion is a complex two-two-quantic curve.

Curves *A* show plainly that the first emulsion with the smaller total grain surface develops to a higher gamma and maximum density than the second emulsion with 50 per cent. greater total grain surface. However, as shown by Curves *B*, the second emulsion when sensitized has a higher gamma and maximum density than the first emulsion sensitized. On comparing the two sets of characteristic curves with each other, it is difficult to believe that decisive differences in development produce these different sensitizing results, in spite of the fact that in these emulsions we are dealing with reactions in different heterogeneous systems. Regardless of how complicated and different the sensitizing reactions may be, sensitivity specks will be produced having different size-distributions according to the same probability laws. This may explain the results of such different sensitivity distributions, which are shown in Figs. 3 and 4.

ROCHESTER 4, NEW YORK,
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LIGHT DISTRIBUTIONS IN ROOMS.

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A few unofficial terms used in the paper are as follows:

pharos F = luminous flux (lumen),
pharosage D = luminous flux density (lumen m^{-2}),
helios H = generalized brightness:

$$H = \pi \lim_{\omega \to 0} \frac{D}{\omega}.$$

If D is in lumens per square meter and ω is in steradians, then H is expressed in *blondels*.

The above names are proposed international words, designed to reduce ambiguity in scientific terminology and to do their humble bit toward world understanding and cooperation.*

I. INTRODUCTION.

Integral-equation theory makes possible the determination of helios distribution in artificially lighted rooms, taking into account the infinitude of light reflections among the various surfaces. Obviously, this subject is of fundamental importance in lighting design. The helios distribution is the basic factor that determines whether visual conditions are satisfactory or not. Also, a knowledge of the helios distribution allows the calculation of incident pharosage (lumen m^{-2}) at any point in the room and on surfaces tilted at any angle.

The general equations for helios distributions in rooms were developed [1] in 1941. The case of the rectangular cylinder was checked against experimental data on lightwells, and the agreement was found to be very satisfactory.[2] The application of the general equations to room lighting was also made.[3]

A peculiarity of the subject is the large number of independent variables: the helios distribution depends on room shape, on distribution

* See "A system of photometric concepts," *J. Opt. Soc. Am.*, **32**, 1942, p. 348; "The names of physical concepts," *Am. J. Phys.*, **10**, 1942, p. 134; "International names in colorimetry," *J. Opt. Soc. Am.*, **36**, 1946; "Internationality in the names of scientific concepts," *Am. J. Phys.*, **14**, 1946.

[1] Parry Moon, *J. Opt. Soc. Am.*, **31**, 223 (1941).

[2] W. T. White, *J. Opt. Soc. Am.*, **31**, 308 (1941); Parry Moon, *J. Opt. Soc. Am.*, **31**, 301 (1941).

[3] Parry Moon, *J. Opt. Soc. Am.*, **31**, 374 (1941); **31**, 723 (1941).

of light from the luminaires, and on three independent reflectances. As a result, the equations are inherently cumbersome; and for routine design, tables or graphs are almost a necessity. Tables were computed by means of the previous equations,[1] and these tables are presented here. In making the computations, we found it convenient to change the notation slightly and to employ the room index to a greater extent than in previous papers. Thus it has seemed advisable to repeat the previous equations in the new notation in order that the tables may be understood with a minimum of effort.

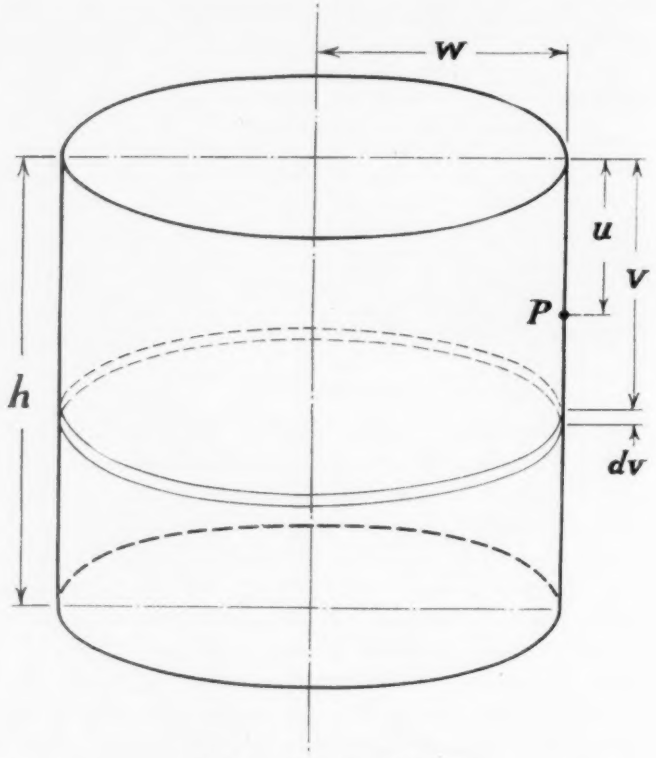

FIG. 1. Circular cylinder.

2. KERNELS FOR CYLINDERS OF VARIOUS CROSS SECTIONS.

The integral equation for a right cylinder *of any cross section*, without top or bottom, is

$$H_1(s) = H_{01}(s) + \rho_1 \int_0^{h/w} K_{11}(s, t) H_1(t) dt. \tag{1}$$

This equation expresses the helios $H_1(s)$ that resuits from an infinite number of reflections of light within the cylinder. Here $H_{01}(s)$ is the initial helios distribution (no interflections) produced by the light sources, and ρ_1 is the reflectance of the inside of the cylinder.

The kernel $K_{11}(s, t)$ of the integral equation is different for each different cross section of cylinder. Physically, the kernal represents the pharosage produced by an infinitesimal luminous element, per unit helios, per unit width of element. Thus the kernel is obtained by any of the well-known methods of calculating pharosage.

Consider the *right-circular cylinder* of Fig. 1. The pharosage dD (lumen m^{-2}) on the inside of the cylinder at P, produced by a luminous ring at v having the helios $H_1(v)$, is

$$dD = \frac{H_1(v)}{2w}\left[1 - \frac{|v-u|[(v-u)^2 + 6w^2]}{[(v-u)^2 + 4w^2]^{3/2}}\right]dv, \qquad (2)$$

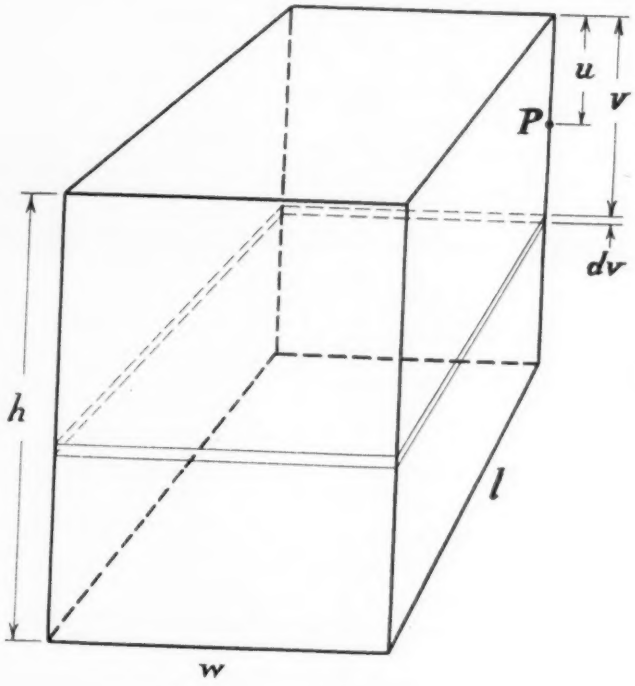

FIG. 2. Rectangular cylinder.

where $H_1(v)$ is in blondels and all dimensions are in the same units (meters, for instance). It is convenient to introduce the normalized variables,

$$s = u/w, \qquad t = v/w. \qquad (3)$$

Then equation (2) becomes

$$dD = \frac{H_1(t)}{2}\left[1 - \frac{|s-t|[(s-t)^2 + 6]}{[(s-t)^2 + 4]^{3/2}}\right]dt. \qquad (2a)$$

The kernel [4] of the integral equation is equal to the pharosage at P, per unit helios, per unit t. Thus the kernel for the *right circular cylinder* is

$$K_{11}(s,t) = \frac{1}{2}\left[1 - \frac{|s-t|[(s-t)^2+6]}{[(s-t)^2+4]^{3/2}}\right]. \tag{4}$$

A right cylinder of rectangular section is shown in Fig. 2. The pharosage at P is no longer entirely independent of position around the cylinder, but its average value at a given depth u is still obtainable by elementary methods.[4] For a luminous strip of width dv and helios $H_1(v)$, the average pharosage at depth u is

$$dD = \frac{H_1(v)}{\pi(w+l)}\left\{\ln\left[\frac{l^2+(u-v)^2}{(u-v)^2}\right] - \ln\left[\frac{w^2+l^2+(u-v)^2}{w^2+(u-v)^2}\right] + wl\left[\frac{w}{[w^2+(u-v)^2]^{3/2}}\tan^{-1}\frac{l}{[w^2+(u-v)^2]^{1/2}} + \frac{l}{[l^2+(u-v)^2]^{3/2}}\tan^{-1}\frac{w}{[l^2+(u-v)^2]^{1/2}}\right]\right\}. \tag{5}$$

Using the normalized variables of equation (3), one obtains the kernel.

$$K_{11}(s,t) = \frac{1}{\pi(1+l/w)}\left\{\ln\left[\frac{(l/w)^2+(s-t)^2}{(s-t)^2}\right] - \ln\left[\frac{1+(l/w)^2+(s-t)^2}{1+(s-t)^2}\right] + \left(\frac{l}{w}\right)\left\{\frac{1}{[1+(s-t)^2]^{3/2}}\tan^{-1}\frac{(l/w)}{[1+(s-t)^2]^{1/2}} + \frac{(l/w)}{[(l/w)^2+(s-t)^2]^{3/2}}\tan^{-1}\frac{1}{[(l/w)^2+(s-t)^2]^{1/2}}\right\}\right\}. \tag{6}$$

For a square cylinder, $l/w = 1$ and equation (6) reduces to

$$K_{11}(s,t) = \frac{1}{\pi}\left\{\ln\left[\frac{1+(s-t)^2}{(s-t)[2+(s-t)^2]^{1/2}}\right] + \frac{1}{[1+(s-t)^2]^{3/2}}\tan^{-1}\frac{1}{[1+(s-t)^2]^{1/2}}\right\}. \tag{7}$$

For two parallel walls of finite height h but of infinite length l, equation (6) becomes

$$K_{11}(s,t) = \tfrac{1}{2}[1+(s-t)^2]^{-3/2}. \tag{8}$$

These kernels are in agreement with those given previously.[4]

[4] Parry Moon, *J. Opt. Soc. Am.*, **30**, 195 (1940).

3. ROOM INDEX.

There appears to be no known way of obtaining a rigorous solution of the integral equation with any of the foregoing kernels. Buckley has shown,[5] however, that the kernel for the circular cylinder can be approximated by an exponential function, in which case equation (1) is easily solved. We propose to use the same kind of approximation for cylinders of all cross sections. The exact kernel for the circular cylinder is plotted in Fig. 3, using equation (4). The exponential approximation is indicated by the straight line. A somewhat similar curve (dotted) for infinitely long walls ($l \to \infty$, equation (8)) will be approximated by the same straight line. Other curves and their approximations are shown in Figs. 4 to 6, inclusive. A better fit can be obtained by using the sum of several exponentials, as has been done by Buckley and by Yamauti for the circular cylinder.[5] Such a representation, however, complicates the analysis so much that it is impracticable for our purpose. It would almost seem that one must choose between an approximate treatment of room lighting and no treatment at all. We have chosen the former alternative.

The kernels of Figs. 3 to 6 are approximated by expressions of the form,

$$K_{11}(s, t) = Ae^{-a|s-t|}, \tag{9}$$

where A and a are constants. It was discovered in 1941 that these constants could be expressed in terms of the perimeter and cross-sectional area of the cylinder [1]:

$$a = \frac{pw}{2S}, \qquad A = a/2, \tag{10}$$

where p = perimeter of a horizontal cross section,
S = area of the cross section,
w = one of the dimensions of the cylinder, such as radius or length of side.

Figures 3 to 6, where the straight lines are determined by equations (9) and (10), show that the true kernels for a variety of shapes are approximated by the foregoing simple expressions. It seems reasonable to assume that equations (9) and (10) will apply to many other shapes.

On the basis of the assumption, we can say that the integral equation for a right cylinder of given cross section has the solution,[1]

$$H_1(s)/H_{01} = \frac{1}{1 - \rho_1} + B_1 \cosh ks + C_1 \sinh ks, \tag{11}$$

[5] H. Buckley, *Phil. Mag.*, **4**, 753 (1927); **6**, 447 (1928). Z. Yamauti, *Res. El. Lab.*, Tokyo, No. 378, 1934 (in Japanese).

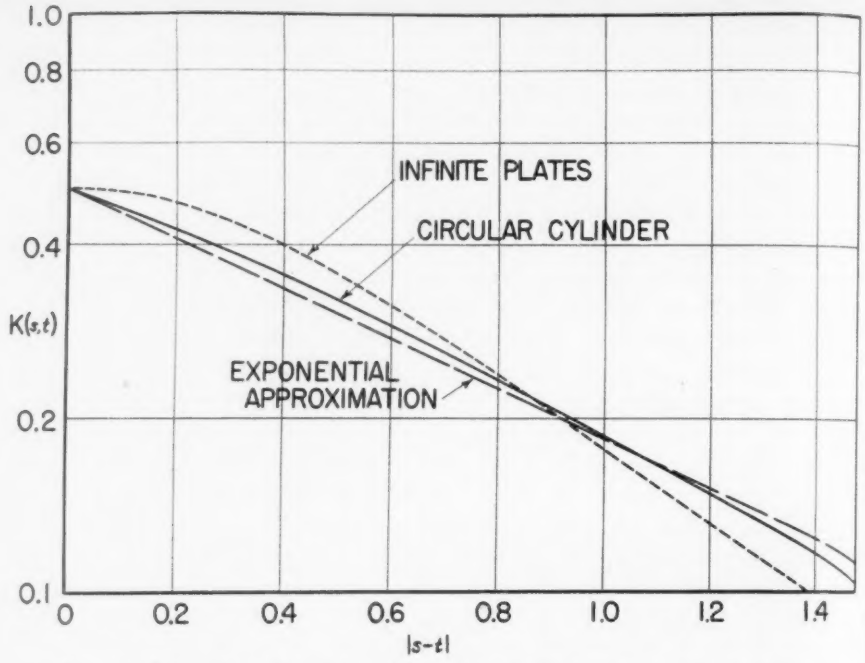

FIG. 3. Comparison of true kernels and their approximation by means of an exponential function.

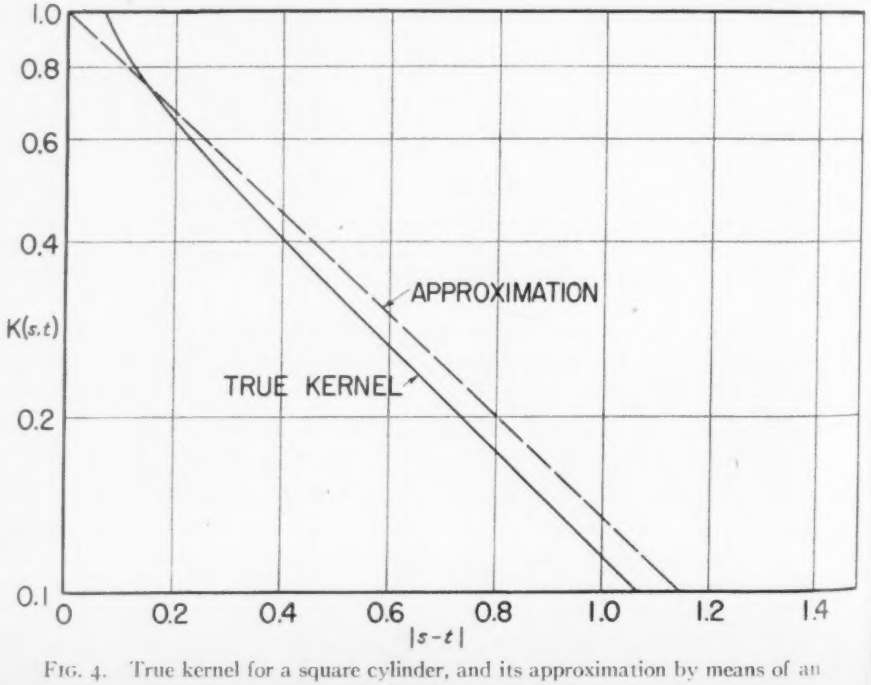

FIG. 4. True kernel for a square cylinder, and its approximation by means of an exponential function, $w/l = 1.00$.

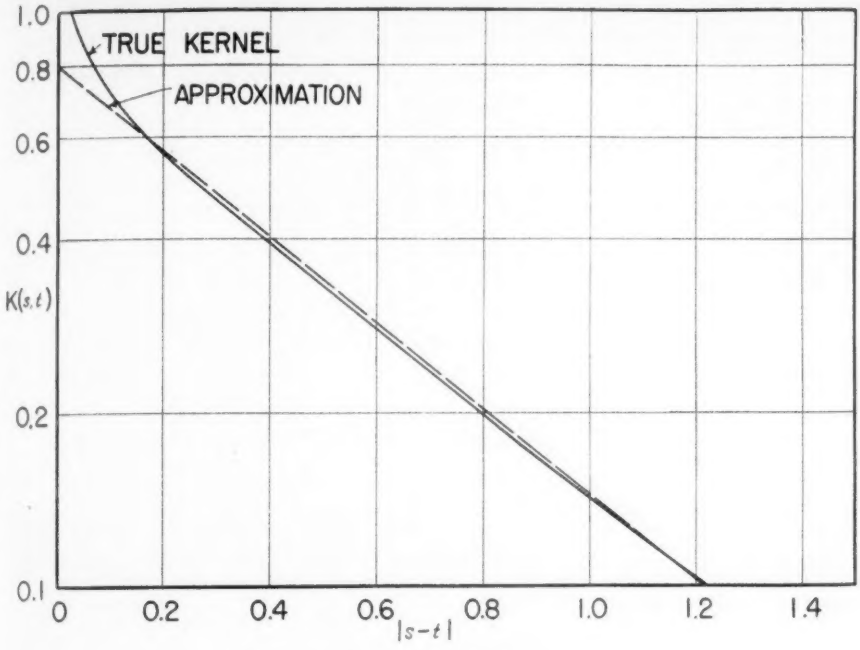

FIG. 5. Rectangular cylinder, $w/l = 2.0$.

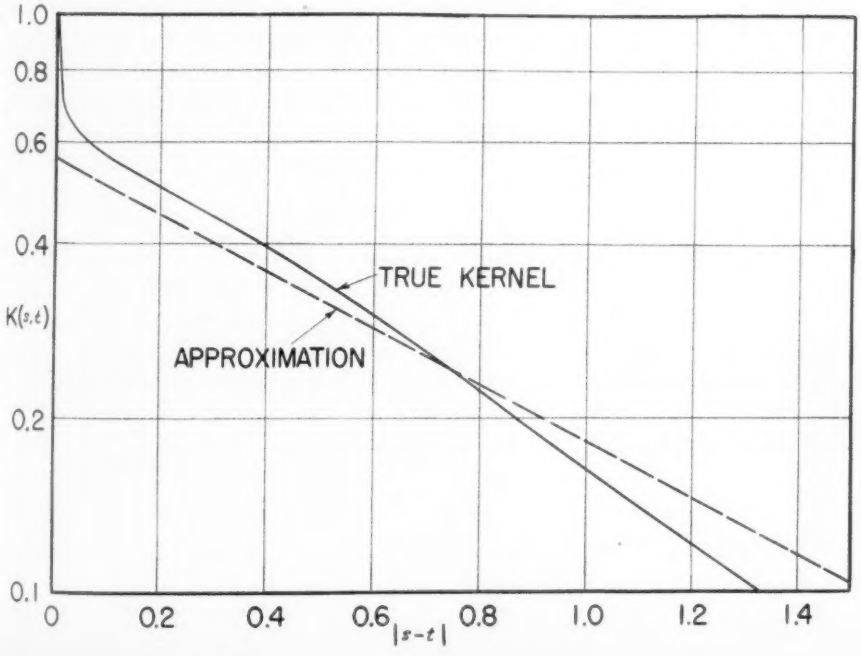

FIG. 6. Rectangular cylinder, $w/l = 8.0$.

where $k = a(1 - \rho_1)^{1/2}$,

$$B_1 = -\frac{\rho_1 E}{1 - \rho_1}\left[\sinh(kh/w) + (1 - \rho_1)^{1/2}\cosh(kh/w) + (1 - \rho_1)^{1/2}\right],$$

$$C_1 = \frac{\rho_1 E}{1 - \rho_1}\left[\cosh(kh/w) + (1 - \rho_1)^{1/2}\sinh(kh/w) - 1\right], \tag{12}$$

$$E = \{(2 - \rho_1)\sinh(kh/w) + 2(1 - \rho_1)^{1/2}\cosh(kh/w)\}^{-1}.$$

Notice that a appears only in the products as and ah/w. For a rectangular cylinder,

$$p = 2(l + w), \qquad S = lw, \qquad s = u/w.$$

Thus, by equation (10),

$$ks = \frac{2(l + w)w}{2lw}\frac{u}{w}(1 - \rho_1)^{1/2} = 2(1 - \rho_1)^{1/2}\left[\frac{h(l + w)}{2lw}\right]\frac{u}{h},$$

$$kh/w = 2(1 - \rho_1)^{1/2}\left[\frac{h(l + w)}{2lw}\right].$$

But the quantity in brackets is the *room index of Hisano.*[6] Thus for any rectangular cylinder,

$$ks = 2k_r(1 - \rho_1)^{1/2}u/h, \tag{13}$$
$$kh/w = 2k_r(1 - \rho_1)^{1/2}.$$

The room index k_r is widely used in lighting design, so equation (13) is a convenient way of expressing the results of integral-equation theory for room lighting. The helios distribution in any rectangular cylinder is then expressed by equations (11) and (12), with equation (13) substituted. Hisano's formula for the room index of a rectangular room,

$$k_r = \frac{h(l + w)}{2lw}, \tag{14}$$

was obtained empirically by the examination of a large amount of experimental data.[6] It is here correlated with the theoretical treatment of integral-equation theory.[1]

Hisano's equation (14) applies only to rectangular rooms. The success in the application of equation (9) to other cross sections, however, suggests the possibility of extending the concept of room index to other shapes. From equations (10) and (13),

$$ks = 2k_r(1 - \rho_1)^{1/2}u/h = a(1 - \rho_1)^{1/2}u/w = \frac{pw}{2S}(1 - \rho_1)^{1/2}u/w,$$

so *the general expression for room index is*

$$k_r = \frac{ph}{4S}. \tag{15}$$

[6] K. Hisano, *Res. El. Lab.*, Tokyo, No. 394, 1936 (in Japanese). Translated into English by Harry Shiramizu, edited by Parry Moon, Illum. Eng. **41**, 232 (1946).

Equation (15) may be employed in calculating the room index for a right cylinder of any cross section without top or bottom. The helios distribution is

$$H_1(u)/H_{01} = \frac{1}{1-\rho_1} + B_1 \cosh\left[2k_r(1-\rho_1)^{1/2}u/h\right] + C_1 \sinh\left[2k_r(1-\rho_1)^{1/2}u/h\right] \quad (11a)$$

where B_1 and C_1 are obtained by substituting equation (13) in equation (12). If, instead of a cylinder, one has a room with reflecting ceiling and floor, the same kernels are used and thus the same formula for room index continues to apply. This case will be considered in the next section.

TABLE I.

Typical Room Indices.

l/w	p	S	a	A	k_r
Rectangles (w = shortest side)					
1.0	$4w$	w^2	2.000	1.000	$1.000\ h/w$
1.5	$5w$	$1.5\ w^2$	1.667	0.833	$0.833\ h/w$
2.0	$6w$	$2\ w^2$	1.500	0.750	$0.750\ h/w$
3.0	$8w$	$3\ w^2$	1.333	0.667	$0.667\ h/w$
4.0	$10w$	$4\ w^2$	1.250	0.625	$0.625\ h/w$
8.0	$18w$	$8\ w^2$	1.125	0.562	$0.562\ h/w$
∞	$2l$	$l\ w$	1.000	0.500	$0.500\ h/w$
Right Triangles (w = shortest side)					
1.0	$3.414\ w$	$0.50\ w^2$	3.414	1.707	$1.707\ h/w$
2.0	$5.24\ w$	$1.00\ w^2$	2.620	1.310	$1.310\ h/w$
3.0	$7.16\ w$	$1.50\ w^2$	2.38	1.19	$1.19\ h/w$
4.0	$9.12\ w$	$2.00\ w^2$	2.28	1.14	$1.14\ h/w$
∞	$2l$	$w\ l/2$	2.00	1.00	$1.00\ h/w$
Regular Polygons (w = side)					
$n = 3$	$3w$	$0.433\ w^2$	3.46	1.73	$1.73\ h/w$
5	$5w$	$1.720\ w^2$	1.45	1.22	$1.22\ h/w$
6	$6w$	$2.598\ w^2$	1.155	0.577	$0.577\ h/w$
8	$8w$	$4.828\ w^2$	0.829	0.415	$0.415\ h/w$
Circle (w = radius = l)					
1.0	$2\pi w$	πw^2	1.000	0.500	$0.500\ h/w$
Ellipses (w = minor semi-axis, l = major semi-axis)					
1.5	$7.95\ w$	$1.5\pi w^2$	0.842	0.421	$0.421\ h/w$
2.0	$9.70\ w$	$2\pi w^2$	0.771	0.385	$0.385\ h/w$
3.0	$13.36\ w$	$3\pi w^2$	0.706	0.353	$0.353\ h/w$
∞	$4l$	$\pi l\ w$	0.635	0.317	$0.317\ h/w$

For a rectangular room, the general formula, equation (15), reduces to Hisano's formula, equation (14). For square rooms,

$$k_r = h/w \tag{14a}$$

and for the cubical room,

$$k_r = 1.00.$$

It is sometimes helpful to have values of a, A, and k_r for non-rectangular rooms with flat ceilings. Such data are given in Table I.

4. CYLINDER WITH ENDS.

The right cylinders of Section 3 can be fitted with plane ends to give results that are applicable to room lighting. Assume that the helios is uniform over the entire ceiling and that the helios has another constant value for the floor. Use subscript 2 for ceiling and 3 for floor, and let

$K_{12}(s)$, $K_{13}(s)$ = average pharosage (lumen/m^2) produced on ceiling (or floor) by a luminous band at s, per unit helios and per unit width of band;

$K_{21}(s)$, $K_{31}(s)$ = average pharosage produced on wall at s by luminous ceiling (or floor), per unit helios;

K_{32} = average pharosage on ceiling, produced by floor of unit helios;

K_{23} = average pharosage on floor, produced by ceiling of unit helios.

Then it is evident that

$$\begin{aligned} H_1(s) &= H_{01} + \rho_1 \left\{ \int_0^{h/w} K_{11}|s-t| H_1(t)dt \right. \\ &\qquad \left. + K_{21}(s)\cdot H_2 + K_{31}(s)\cdot H_3 \right\}, \\ H_2 &= H_{02} + \rho_2 \left\{ \int_0^{h/w} K_{12}(t) H_1(t)dt + K_{32}H_3 \right\}, \\ H_3 &= H_{03} + \rho_3 \left\{ \int_0^{h/w} K_{13}(t) H_1(t)dt + K_{23}H_2 \right\}. \end{aligned} \tag{16}$$

On the basis of the approximate kernel,[1]

$$\begin{aligned} K_{12}(s) &= ae^{-as}, \\ K_{21}(s) &= \tfrac{1}{2}e^{-as}, \\ K_{13}(s) &= ae^{-a(h/w-s)}, \\ K_{31}(s) &= \tfrac{1}{2}e^{-a(h/w-s)}, \\ K_{23} &= K_{32} = e^{-ah/w}. \end{aligned} \tag{17}$$

Solution of the integral equation (16) was given previously [1] for the general case of arbitrary values of H_{01}, H_{02}, H_{03} applied simultaneously to walls, ceilings, and floor. For the present purpose, it is more convenient to calculate separately the three special cases of ceiling lighting,

wall lighting, and floor lighting. Solutions of equation (16) for these three conditions are listed below.[7]

Wall Lighting ($H_{02} = H_{03} = 0$, $H_{01} \neq 0$).

The wall helios is given by the expression,

$$H_1(u)/H_{01} = \frac{1}{1-\rho_1} + B_1 \cosh\left[2k_r(1-\rho_1)^{1/2}u/h\right] + C_1 \sinh\left[2k_r(1-\rho_1)^{1/2}u/h\right]. \quad (18)$$

Here

$$B_1 = -\frac{\rho_1 E}{1-\rho_1}\left[(1-\rho_2)(1-\rho_3)\sinh\left[2k_r(1-\rho_1)^{1/2}\right] + (1-\rho_1)^{1/2}(1-\rho_2)(1+\rho_3)\cosh\left[2k_r(1-\rho_1)^{1/2}\right] + (1-\rho_1)^{1/2}(1+\rho_2)(1-\rho_3)\right],$$

$$C_1 = \frac{\rho_1(1-\rho_2)E}{1-\rho_1}\left[(1-\rho_3)\cosh\left[2k_r(1-\rho_1)^{1/2}\right] + (1-\rho_1)^{1/2}(1+\rho_3)\sinh\left[2k_r(1-\rho_1)^{1/2}\right] - (1-\rho_3)\right], \quad (19)$$

$$E = \left\{\left[2(1+\rho_2\rho_3) - \rho_1(1+\rho_2)(1+\rho_3)\right]\sinh\left[2k_r(1-\rho_1)^{1/2}\right] + 2(1-\rho_1)^{1/2}(1-\rho_2\rho_3)\cosh\left[2k_r(1-\rho_1)^{1/2}\right]\right\}^{-1}.$$

The ceiling helios is

$$H_2/H_{01} = \frac{2\rho_2 E}{1-\rho_1}\left[(1-\rho_1)(1+\rho_3)\sinh\left[2k_r(1-\rho_1)^{1/2}\right] + (1-\rho_1)^{1/2}(1-\rho_3)\cosh\left[2k_r(1-\rho_1)^{1/2}\right] - (1-\rho_1)^{1/2}(1-\rho_3)\right]. \quad (20)$$

The floor helios is

$$H_3/H_{01} = \frac{2\rho_3 E}{1-\rho_1}\left[(1-\rho_1)(1+\rho_2)\sinh\left[2k_r(1-\rho_1)^{1/2}\right] + (1-\rho_1)^{1/2}(1-\rho_2)(\cosh\left[2k_r(1-\rho_1)^{1/2}\right] - 1)\right]. \quad (21)$$

The average pharosage (lumen m^{-2}) on an imaginary horizontal surface at distance h' below the ceiling is obtained by adding the effects of ceiling and upper walls. As in the previous paper,[1]

$$D_{AV}/H_{01} = \frac{1}{1-\rho_1} + \frac{E}{1-\rho_1} \times \left\{\left[-(1-\rho_2)\left[2\rho_3 - \rho_1(1+\rho_3)\right]\cosh\left[2k_r(1-\rho_1)^{1/2}\right] + 2\rho_3(1-\rho_1)^{1/2}(1-\rho_2)\sinh\left[2k_r(1-\rho_1)^{1/2}\right] + (1-\rho_3)\left[2\rho_2 - \rho_1(1+\rho_2)\right]\right]\sinh\left[2k_r(1-\rho_1)^{1/2}h'/h\right] - \left[(1-\rho_2)\left[2-\rho_1(1+\rho_3)\right]\sinh\left[2k_r(1-\rho_1)^{1/2}\right] + 2(1-\rho_1)^{1/2}(1-\rho_2)\cosh\left[2k_r(1-\rho_1)^{1/2}\right] + 2\rho_2(1-\rho_1)^{1/2}(1-\rho_3)\right]\cosh\left[2k_r(1-\rho_1)^{1/2}h'/h\right]\right\}. \quad (22)$$

[7] Taken from Reference 1, p. 225 to 227. A re-check of these equations revealed one typographical error: in Eq. (13b) of p. 227, last term, the first u_2 should be v_2.

Ceiling Lighting ($H_{01} = H_{03} = 0$, $H_{02} \neq 0$).

$$H_1(u)/H_{02} = B_2 \cosh [2k_r(1 - \rho_1)^{1/2}u/h] + C_2 \sinh [2k_r(1 - \rho_1)^{1/2}u/h] \quad (23)$$

where

$$B_2 = \rho_1 E[(1 - \rho_3) \sinh [2k_r(1 - \rho_1)^{1/2}] + (1 - \rho_1)^{1/2}(1 + \rho_3) \cosh [2k_r(1 - \rho_1)^{1/2}]], \quad (24)$$

$$C_2 = -\rho_1 E[(1 - \rho_3) \cosh [2k_r(1 - \rho_1)^{1/2}] + (1 - \rho_1)^{1/2}(1 + \rho_3) \sinh [2k_r(1 - \rho_1)^{1/2}]].$$

$$H_2/H_{02} = E\{(2 - \rho_1(1 + \rho_3)) \sinh [2k_r(1 - \rho_1)^{1/2}] + 2(1 - \rho_1)^{1/2} \cosh [2k_r(1 - \rho_1)^{1/2}]\}. \quad (25)$$

$$H_3/H_{02} = 2\rho_3(1 - \rho_1)^{1/2}E. \quad (26)$$

TABLE II.

Wall Lighting, $-B_1$.

k_r	ρ_2=0.80 (Ceiling)				ρ_2=0.70			ρ_2=0.50		
	ρ_1=0.80	0.50	0.30	0.10	0.50	0.30	0.10	0.50	0.30	0.10
					ρ_3=0.10 (Floor)					
0	4.000	1.0000	^{0}4286	^{0}1111	1.0000	^{0}4286	^{0}1111	1.0000	^{0}4286	^{0}1111
0.1	3.825	^{0}8973	3694	^{1}9213	^{0}9043	3733	^{1}9332	^{0}9177	3808	^{1}9566
0.2	3.647	8047	3196	^{1}7718	8182	3267	^{1}7927	8441	3405	^{1}8338
0.3	3.469	7218	2778	^{1}6528	7413	2876	^{1}6805	7786	3066	^{1}7351
0.4	3.295	6481	2425	^{1}5567	6732	2546	^{1}5898	7209	2780	^{1}6551
0.5	3.125	5830	2127	^{1}4785	6130	2267	^{1}5162	6700	2538	^{1}5902
0.7	2.803	4755	1664	^{1}3627	5136	1833	^{1}4066	5860	2163	^{1}4933
1.0	2.381	3595	^{0}1205	^{1}2552	4065	^{0}1404	^{1}3049	4955	1790	^{1}4037
2.0	1.475	1906	^{1}6335	^{1}1367	2504	^{1}8723	^{1}1930	3632	1332	^{1}3053
					ρ_3=0.30 (Floor)					
0	4.000	1.0000	^{0}4286	^{0}1111	1.0000	^{0}4286	^{0}1111	1.0000	^{0}4286	^{0}1111
0.1	3.761	^{0}8624	^{0}3503	^{1}8631	^{0}8744	^{0}3568	^{1}8827	^{0}8961	3687	^{1}9192
0.2	3.534	7514	2927	^{1}6953	7722	3032	^{1}7252	8102	3228	^{1}7821
0.3	3.322	6604	2487	^{1}5745	6879	2619	^{1}6107	7388	2869	^{1}6803
0.4	3.123	5847	2141	^{1}4839	6176	2293	^{1}5241	6788	2583	^{1}6026
0.5	2.937	5212	1864	^{1}4137	5585	2031	^{1}4572	6281	2352	^{1}5421
0.7	2.605	4217	1454	^{1}3143	4658	1643	^{1}3623	5484	2009	^{1}4566
1.0	2.194	3204	^{0}1068	^{1}2263	3713	1279	^{1}2783	4672	1686	^{1}3811
2.0	1.377	1801	^{1}6065	^{1}1322	2408	^{1}8442	^{1}1888	3554	1310	^{1}3010

NOTE.—Superscripts refer to number of zeros after decimal point. When no superscript is used, a zero is understood. For example, 1906 = 0.1906, ^{1}6335 = 0.06335.

$$D_{AV}/H_{02} = \frac{H_2}{H_{02}} \cosh\left[2k_r(1-\rho_1)^{1/2}h'/h\right] - E\{(2-\rho_1(1+\rho_3))\cosh\left[2k_r(1-\rho_1)^{1/2}\right] + 2(1-\rho_1)^{1/2}\sinh\left[2k_r(1-\rho_1)^{1/2}\right]\}\sinh\left[2k_r(1-\rho_1)^{1/2}h'/h\right]. \quad (27)$$

Floor Lighting ($H_{01} = H_{02} = 0$, $H_{03} \neq 0$).

$$H_1(u)/H_{03} = B_3\cosh\left[2k_r(1-\rho_1)^{1/2}u/h\right] + C_3\sinh\left[2k_r(1-\rho_1)^{1/2}u/h\right], \quad (28)$$

where

$$B_3 = \rho_1(1-\rho_1)^{1/2}(1+\rho_2)E, \quad (29)$$
$$C_3 = \rho_1(1-\rho_2)E.$$

$$H_2/H_{03} = 2\rho_2(1-\rho_1)^{1/2}E. \quad (30)$$

$$H_3/H_{03} = E\{(2-\rho_1)(1+\rho_2)\sinh\left[2k_r(1-\rho_1)^{1/2}\right] + 2(1-\rho_1)^{1/2}\cosh\left[2k_r(1-\rho_1)^{1/2}\right]\}. \quad (31)$$

$$D_{AV}/H_{03} = E\{-(2\rho_2-\rho_1(1+\rho_2))\sinh\left[2k_r(1-\rho_1)^{1/2}h'/h\right] + 2\rho_2(1-\rho_1)^{1/2}\cosh\left[2k_r(1-\rho_1)^{1/2}h'/h\right]\}. \quad (32)$$

TABLE III.

Wall Lighting, C_1.

k_r	ρ_2=0.80 (Ceiling)				ρ_2=0.70			ρ_2=0.50		
	ρ_1=0.80	0.50	0.30	0.10	0.50	0.30	0.10	0.50	0.30	0.10
					ρ_3=0.10 (Floor)					
0	0	0	0	0	0	0	0	0	0	0
0.1	$^{1}4339$	$^{1}1613$	$^{2}7861$	$^{2}2224$	$^{1}2389$	$^{1}1167$	$^{2}3309$	$^{1}3882$	$^{1}1905$	$^{2}5427$
0.2	$^{1}8761$	$^{1}3070$	$^{1}1447$	$^{2}3973$	$^{1}4538$	$^{1}2148$	$^{2}5922$	$^{1}7351$	$^{1}3510$	$^{2}9747$
0.3	$^{0}1318$	$^{1}4372$	$^{1}2003$	$^{2}5367$	$^{1}6456$	$^{1}2974$	$^{2}8009$	$^{0}1043$	$^{1}4860$	$^{1}1321$
0.4	1752	$^{1}5530$	$^{1}2471$	$^{2}6492$	$^{1}8157$	$^{1}3670$	$^{2}9696$	1316	$^{1}6000$	$^{1}1602$
0.5	2175	$^{1}6552$	$^{1}2867$	$^{2}7408$	$^{1}9658$	$^{1}4258$	$^{1}1114$	1556	$^{1}6961$	$^{1}1831$
0.7	2974	$^{1}8242$	$^{1}3482$	$^{2}8768$	$^{0}1214$	$^{1}5172$	$^{1}1311$	1952	$^{1}8458$	$^{1}2171$
1.0	4023	$^{0}1007$	$^{1}4091$	$^{1}1003$	1481	$^{1}6079$	$^{1}1499$	2378	$^{1}9942$	$^{1}2485$
2.0	6274	1272	$^{1}4848$	$^{1}1135$	1871	$^{1}7205$	$^{1}1706$	3001	$^{0}1179$	$^{1}2829$
					ρ_3=0.30 (Floor)					
0	0	0	0	0	0	0	0	0	0	0
0.1	$^{1}5949$	$^{1}2161$	$^{1}1040$	$^{2}2905$	$^{1}3135$	$^{1}1514$	$^{2}4247$	$^{1}4900$	$^{1}2385$	$^{2}6739$
0.2	$^{0}1157$	$^{1}3906$	$^{1}1804$	$^{2}4870$	$^{1}5686$	$^{1}2644$	$^{2}7178$	$^{1}8946$	$^{1}4214$	$^{1}1156$
0.3	1685	$^{1}5337$	$^{1}2389$	$^{2}6284$	$^{1}7788$	$^{1}3515$	$^{2}9310$	$^{0}1232$	$^{1}5645$	$^{1}1514$
0.4	2180	$^{1}6525$	$^{1}2848$	$^{2}7348$	$^{1}9542$	$^{1}4203$	$^{1}1092$	1514	$^{1}6786$	$^{1}1787$
0.5	2641	$^{1}7524$	$^{1}3216$	$^{2}8168$	$^{0}1102$	$^{1}4755$	$^{1}1217$	1753	$^{1}7706$	$^{1}1999$
0.7	3466	$^{1}9087$	$^{1}3760$	$^{2}9331$	1333	$^{1}5573$	$^{1}1393$	2129	$^{1}9074$	$^{1}2301$
1.0	4486	$^{0}1068$	$^{1}4273$	$^{1}1036$	1569	$^{1}6342$	$^{1}1549$	2512	$^{0}1036$	$^{1}2565$
2.0	6517	1288	$^{1}4886$	$^{1}1141$	1895	$^{1}7259$	$^{1}1714$	3039	1187	$^{1}2842$

These equations make possible the calculation of the average helios of the ceiling and the floor, as well as the helios distribution on the walls of the room. The room must have a flat ceiling but it may have a wide variety of horizontal cross sections, the room index k_r being obtained by the method of Section 3. The equations for wall lighting are based on the assumption that all the light from the luminaires is incident on the walls, resulting in an average wall helios (without interflections) of H_{01}. Similarly, the equations for ceiling lighting and

TABLE IV.

Wall Lighting H_{1T}/H_{01}.

k_r'	$\rho_2=0.80$ (Ceiling)				$\rho_2=0.70$			$\rho_2=0.50$		
	$\rho_1=0.80$	0.50	0.30	0.10	0.50	0.30	0.10	0.50	0.30	0.10
					$\rho_3=0.10$ (Floor)					
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.1	1.1746	1.1027	1.0592	1.0190	1.0957	1.0553	1.0178	1.0823	1.0478	1.0155
0.2	1.3526	1.1953	1.1089	1.0339	1.1818	1.1019	1.0318	1.1560	1.0881	1.0277
0.3	1.5304	1.2783	1.1508	1.0458	1.2587	1.1410	1.0431	1.2214	1.1220	1.0376
0.4	1.7052	1.3519	1.1861	1.0554	1.3269	1.1740	1.0521	1.2791	1.1506	1.0456
0.5	1.8755	1.4170	1.2158	1.0633	1.3870	1.2019	1.0595	1.3301	1.1747	1.0521
0.7	2.197	1.5246	1.2622	1.0748	1.4864	1.2452	1.0705	1.4141	1.2123	1.0618
1.0	2.619	1.6405	1.3081	1.0856	1.5935	1.2882	1.0806	1.5045	1.2495	1.0707
2.0	3.525	1.8095	1.3652	1.0974	1.7496	1.3413	1.0917	1.6368	1.2959	1.0806
					$\rho_3=0.30$ (Floor)					
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.1	1.2394	1.1376	1.0783	1.0248	1.1256	1.0718	1.0228	1.1039	1.0599	1.0192
0.2	1.4657	1.2486	1.1359	1.0416	1.2278	1.1254	1.0386	1.1898	1.1058	1.0329
0.3	1.6781	1.3396	1.1799	1.0537	1.3121	1.1667	1.0500	1.2612	1.1417	1.0431
0.4	1.8775	1.4153	1.2145	1.0627	1.3824	1.1993	1.0587	1.3212	1.1703	1.0509
0.5	2.063	1.4788	1.2421	1.0697	1.4415	1.2254	1.0654	1.3719	1.1934	1.0569
0.7	2.395	1.5783	1.2831	1.0797	1.5342	1.2642	1.0749	1.4516	1.2277	1.0655
1.0	2.806	1.6796	1.3217	1.0885	1.6287	1.3007	1.0833	1.5328	1.2600	1.0730
2.0	3.623	1.8199	1.3679	1.0979	1.7592	1.3442	1.0921	1.6446	1.2980	1.0809

floor lighting assume that all light from the luminaires is incident on ceiling and floor, respectively. If the luminaires illuminate ceiling, floor, and walls directly, the lighting system is analyzed into its three component systems, and the principle of superposition is employed in obtaining the final values of helios and pharosage.

The foregoing formulas for D_{AV} give the average pharosage on a horizontal surface at any distance h' below the ceiling. This quantity is of fundamental importance in the design of lighting systems. For

rectangular rooms, the *distribution* of pharosage over the horizontal surface can be obtained if desired by considering the walls and ceilings as rectangular windows and using the curves or tables of a previous paper.[8,9]

TABLE V.

Wall Lighting, H_{1M}/H_{01}.

k_T	$\rho_2=0.80$ (Ceiling)				$\rho_2=0.70$			$\rho_2=0.50$		
	$\rho_1=0.80$	0.50	0.30	0.10	0.50	0.30	0.10	0.50	0.30	0.10
					$\rho_3=0.10$ (Floor)					
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.1	1.1728	1.1016	1.0586	1.0188	1.0952	1.0550	1.0177	1.0828	1.0481	1.0155
0.2	1.3459	1.1916	1.1069	1.0333	1.1801	1.1009	1.0315	1.1579	1.0892	1.0281
0.3	1.5169	1.2713	1.1471	1.0443	1.2558	1.1394	1.0426	1.2261	1.1246	1.0384
0.4	1.6840	1.3417	1.1808	1.0539	1.3231	1.1721	1.0516	1.2878	1.1553	1.0471
0.5	1.8461	1.4038	1.2093	1.0614	1.3831	1.2001	1.0591	1.3439	1.1822	1.0543
0.7	2.152	1.5076	1.2544	1.0728	1.4847	1.2449	1.0705	1.4414	1.2266	1.0660
1.0	2.563	1.6241	1.3018	1.0842	1.6012	1.2931	1.0823	1.5580	1.2764	1.0784
2.0	3.534	1.8311	1.3785	1.1015	1.8166	1.3741	1.1007	1.7892	1.3655	1.0990
					$\rho_3=0.30$ (Floor)					
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.1	1.2383	1.1370	1.0779	1.0247	1.1256	1.0718	1.0228	1.1052	1.0606	1.0194
0.2	1.4619	1.2466	1.1348	1.0413	1.2282	1.1256	1.0387	1.1944	1.1083	1.0337
0.3	1.6709	1.3361	1.1780	1.0531	1.3132	1.1673	1.0502	1.2709	1.1469	1.0447
0.4	1.8667	1.4104	1.2121	1.0621	1.3848	1.2006	1.0591	1.3373	1.1789	1.0534
0.5	2.049	1.4730	1.2394	1.0690	1.4460	1.2281	1.0661	1.3955	1.2057	1.0605
0.7	2.375	1.5724	1.2808	1.0792	1.5447	1.2698	1.0765	1.4928	1.2485	1.0714
1.0	2.790	1.6780	1.3222	1.0889	1.6523	1.3128	1.0868	1.6039	1.2945	1.0827
2.0	3.698	1.8570	1.3869	1.1032	1.8421	1.3823	1.1023	1.8140	1.3736	1.1006

5. TABLES FOR WALL LIGHTING.

The equations of Section 4 were used in calculating tables covering the usual range of variables encountered in room lighting. A computing machine was employed, and calculations were made to 4 or 5 figures. The results are given in Table II and subsequent tables.

The constants B_1 and C_1 of Tables II and III are needed in calculating helios distribution for the walls, equation (11). Usually, however, it is sufficient to know the average helios H_{1T} at the top of the wall and helios H_{1M} at the middle of the wall (see Section 9). These

[8] Moon and Spencer, *J. Franklin Institute*, **241**, 195 (1946).
[9] P. C. Magnusson, *J. Math. Phys.*, **21**, 250 (1942).

quantities are given in Tables IV and V. The remaining tables (VI to VIII) give H_2, H_3, and D_{AV}. Linear interpolation can be used to obtain intermediate values with engineering accuracy.

6. TABLES FOR CEILING LIGHTING.

Similar to the tables of Section 5 are those for ceiling lighting. The necessary quantities are listed in Tables IX to XIV.

TABLE VI.

Wall Lighting, H_2/H_{01}.

k_r	$\rho_2=0.80$ (Ceiling)				$\rho_2=0.70$			$\rho_2=0.50$		
	$\rho_1=0.80$	0.50	0.30	0.10	0.50	0.30	0.10	0.50	0.30	0.10
				$\rho_3=0.10$ (Floor)						
0	0	0	0	0	0	0	0	0	0	0
0.1	^{0}1940	^{0}1825	^{0}1754	^{0}1688	^{0}1577	^{0}1519	^{0}1465	^{0}1098	^{0}1063	^{0}1030
0.2	3918	3473	3228	3015	2995	2796	2622	2079	1958	1849
0.3	5893	4947	4468	4073	4261	3871	3546	2951	2711	2507
0.4	7835	6256	5513	4927	5383	4777	4292	3722	3346	3040
0.5	^{0}9729	7413	6396	5622	6374	5542	4900	4401	3883	3473
0.7	1.3299	^{0}9325	7768	6653	8011	6732	5803	5521	4718	4118
1.0	1.7988	1.1384	^{0}9126	7607	^{0}9772	7909	6637	6725	5544	4714
2.0	2.803	1.4378	1.0811	8649	1.2336	9370	7547	8489	6569	5362
				$\rho_3=0.30$ (Floor)						
0	0	0	0	0	0	0	0	0	0	0
0.1	^{0}2661	^{0}2445	^{0}2319	^{0}2205	^{0}2069	^{0}1970	^{0}1881	^{0}1386	^{0}1330	^{0}1279
0.2	5174	4419	4026	3696	3752	3441	3178	2530	2350	2193
0.3	7535	6038	5330	4769	5140	4575	4121	3483	3149	2872
0.4	^{0}9749	7382	6353	5575	6297	5470	4833	4283	3784	3390
0.5	1.1810	^{0}8513	7175	6199	7272	6189	5386	4959	4299	3793
0.7	1.5498	1.0280	8389	7082	^{0}8799	7253	6167	6021	5061	4364
1.0	2.006	1.2079	^{0}9530	7863	1.0352	8253	6857	7103	5776	4866
2.0	2.912	1.4562	1.0891	8688	1.2493	9439	7582	8588	6616	5387

7. TABLES FOR FLOOR LIGHTING.

Tables XV to XX apply to the case of a luminous floor, such as might be obtained by employing a floor of diffusing glass, lighted from below. The tables apply also to *direct-lighting systems* in which the luminaires are louvered so that none of their light reaches the ceiling or walls directly. Then the H_{03} of the tables is

$$H_{03} = \rho_3 F_D/S, \tag{33}$$

where F_D = total pharos (lumens) emitted by the luminaires,
S = floor area (m^2),
ρ_3 = reflectance (numeric) of floor.

Note, however, the average pharosage D_{AV} of Table XX gives the effect of walls and ceiling only, just as in Sections 5 and 6. When a *direct-lighting system* is employed, the tabulated value must be increased by the direct pharosage from the luminaires, which is

$$D_{AV} = F_D/S. \tag{34}$$

TABLE VII.

Wall Lighting, H_3/H_{01}.

k_r	ρ_2=0.80 (Ceiling)				ρ_2=0.70			ρ_2=0.50		
	ρ_1=0.80	0.50	0.30	0.10	0.50	0.30	0.10	0.50	0.30	0.10
					ρ_3=0.10 (Floor)					
0	0	0	0	0	0	0	0	0	0	0
0.1	^{1}3714	^{1}3490	^{1}3354	^{1}3230	^{1}3274	^{1}3155	^{1}3045	^{1}2860	^{1}2769	^{1}2684
0.2	^{1}7043	^{1}6245	^{1}5806	^{1}5426	^{1}5887	^{1}5497	^{1}5156	^{1}5200	^{1}4897	^{1}4627
0.3	^{0}1001	^{1}8415	^{1}7608	^{1}6944	^{1}7969	^{1}7247	^{1}6645	^{1}7119	^{1}6544	^{1}6056
0.4	1264	^{0}1012	^{1}8939	^{1}8006	^{1}9631	^{1}8562	^{1}7708	^{1}8696	^{1}7829	^{1}7121
0.5	1497	1147	^{1}9928	^{1}8758	^{0}1096	^{1}9557	^{1}8478	^{1}9996	^{1}8839	^{1}7925
0.7	1881	1336	^{0}1122	^{1}9683	1288	^{0}1090	^{1}9457	^{0}1196	^{0}1027	^{1}9009
1.0	2299	1497	1220	^{0}1033	1458	1198	^{0}1018	1385	1153	^{1}9892
2.0	2931	1654	1301	1079	1642	1296	^{0}1077	1620	1286	1072
					ρ_3=0.30 (Floor)					
0	0	0	0	0	0	0	0	0	0	0
0.1	^{0}1327	^{0}1219	^{0}1156	^{0}1099	^{0}1120	^{0}1067	^{0}1018	^{1}9411	^{1}9034	^{1}8686
0.2	2482	2118	1930	1772	1966	1803	1666	^{0}1687	^{0}1567	^{0}1463
0.3	3485	2793	2467	2209	2615	2328	2099	2285	2066	1886
0.4	4356	3306	2849	2503	3118	2711	2398	2769	2448	2195
0.5	5112	3699	3125	2706	3513	2995	2612	3164	2746	2425
0.7	6339	4242	3480	2953	4074	3373	2880	3758	3165	2736
1.0	7642	4696	3749	3126	4567	3676	3080	4325	3534	2990
2.0	9583	5145	3975	3255	5107	3960	3247	5036	3929	3233

All the tables give ratios that apply without change to any unit of length, consistently used. In the mks system, the unit of helios is the *blondel* corresponding to one lumen per square meter radiated by a perfectly diffusing surface, and the unit of pharosage is the *lumen per square meter*. In the *cgs* system, all H's may be replaced by brightness in *millilamberts*, in which case D_{AV} will be in *millilumens per square cm.* In the English system H's are replaced by brightness in "footlamberts" and D_{AV} is in lumens per square foot ("footcandles").

8. SUPERPOSITION.

The foregoing tables are for the three special cases of wall lighting, ceiling lighting, and floor lighting. The results apply directly to some actual lighting systems; though in many cases the flux from the luminaires is not confined to a single surface, and the actual system is considered to be composed of two or three of the elementary systems of Section 4.

TABLE VIII.

Wall Lighting, D_{AV}/H_{01}.

k_r	$\rho_2=0.80$ (Ceiling)				$\rho_2=0.70$			$\rho_2=0.50$		
	$\rho_1=0.80$	0.50	0.30	0.10	0.50	0.30	0.10	0.50	0.30	0.10
					$\rho_3=0.10$ (Floor)					
0	0	0	0	0	0	0	0	0	0	0
0.1	⁰3307	⁰3107	⁰2984	⁰2872	⁰2883	⁰2777	⁰2678	⁰2453	⁰2374	⁰2301
0.2	6394	5664	5261	4912	5279	4925	4616	4541	4272	4034
0.3	⁰9261	7765	7008	6384	7269	6597	6039	6321	5801	5361
0.4	1.1905	⁰9491	8355	7460	⁰8923	7909	7100	7842	7043	6390
0.5	1.4340	1.0911	⁰9402	8256	1.0303	⁰8946	7902	⁰9149	8060	7200
0.7	1.8619	1.3048	1.0865	⁰9300	1.2427	1.0442	8985	1.1249	⁰9605	8363
1.0	2.376	1.5090	1.2117	1.0111	1.4535	1.1770	⁰9877	1.3487	1.1095	⁰9414
2.0	3.383	1.7809	1.3509	1.0866	1.7562	1.3382	1.0806	1.7107	1.3148	1.0679
					$\rho_3=0.30$ (Floor)					
0	0	0	0	0	0	0	0	0	0	0
0.1	⁰4016	⁰3689	⁰3497	⁰3325	⁰3349	⁰3189	⁰3043	⁰2733	⁰2622	⁰2521
0.2	⁰7624	6506	5924	5437	5963	5466	5046	4968	4612	4304
0.3	1.0861	⁰8693	7668	6858	8035	7146	6433	6819	6159	5616
0.4	1.3773	1.0416	8955	7851	⁰9699	8416	7430	8368	7387	6610
0.5	1.6381	1.1791	⁰9927	8569	1.1053	⁰9396	8167	⁰9678	8377	7383
0.7	2.083	1.3799	1.1259	⁰9491	1.3086	1.0783	9152	1.1750	⁰9864	8483
1.0	2.598	1.5667	1.2368	1.0205	1.5064	1.2001	⁰9962	1.3928	1.1289	⁰9481
2.0	3.562	1.8120	1.3618	1.0891	1.7876	1.3499	1.0830	1.7392	1.3253	1.0705

Given a room with known reflectances ρ_1, ρ_2, ρ_3 and with luminaires that deliver F_1, F_2, and F_3 lumens to walls, ceiling, and floor, respectively. Then the average values of helios are

$$H_{01} = \rho_1 F_1/S_1, \qquad H_{02} = \rho_2 F_2/S, \qquad H_{03} = \rho_3 F_3/S, \tag{35}$$

where S_1 = total area of walls (m^2),
S = area of floor (m^2).

The helios of the walls is

$$H_1(u) = \left(\frac{H_1(u)}{H_{01}}\right) H_{01} + \left(\frac{H_1(u)}{H_{02}}\right) H_{02} + \left(\frac{H_1(u)}{H_{03}}\right) H_{03}, \quad (36)$$

where the ratios in parentheses are obtained from the tables of Sections 5 to 7, and the values of H_{01}, H_{02}, H_{03} are obtained from equation (35).

TABLE IX.

Ceiling Lighting, $B_2 = H_{1T}/H_{02}$.

k_T	$\rho_2=0.80$ (Ceiling)				$\rho_2=0.70$			$\rho_2=0.50$		
	$\rho_2=0.80$	0.50	0.30	0.10	0.50	0.30	0.10	0.50	0.30	0.10
					$\rho_3=0.10$ (Floor)					
0	$^{0}4782$	$^{0}2989$	$^{0}1793$	$^{1}5978$	$^{0}2957$	$^{0}1774$	$^{1}5914$	$^{0}2895$	$^{0}1737$	$^{1}5789$
0.1	5236	3085	1784	$^{1}5736$	3046	1765	$^{1}5690$	2970	1729	$^{1}5599$
0.2	5633	3159	1776	$^{1}5576$	3114	1758	$^{1}5540$	3026	1723	$^{1}5471$
0.3	5978	3216	1771	$^{1}5468$	3166	1754	$^{1}5440$	3069	1720	$^{1}5384$
0.4	6279	3259	1768	$^{1}5396$	3205	1750	$^{1}5372$	3102	1717	$^{1}5326$
0.5	6535	3292	1765	$^{1}5347$	3235	1748	$^{1}5326$	3127	1715	$^{1}5286$
0.7	6943	3336	1762	$^{1}5291$	3276	1745	$^{1}5274$	3160	1712	$^{1}5240$
1.0	7353	3370	1760	$^{1}5258$	3306	1743	$^{1}5243$	3185	1710	$^{1}5213$
2.0	7856	3394	1759	$^{1}5242$	3327	1742	$^{1}5228$	3203	1709	$^{1}5200$
					$\rho_3=0.30$ (Floor)					
0	$^{0}6842$	$^{0}4276$	$^{0}2566$	$^{1}8553$	$^{0}4113$	$^{0}2468$	$^{1}8228$	$^{0}3823$	$^{0}2294$	$^{1}7647$
0.1	7018	4042	2306	$^{1}7325$	3908	2239	$^{1}7140$	3665	2116	$^{1}6798$
0.2	7167	3873	2135	$^{1}6592$	3758	2086	$^{1}6477$	3548	1995	$^{1}6260$
0.3	7293	3750	2021	$^{1}6133$	3649	1983	$^{1}6057$	3462	1911	$^{1}5910$
0.4	7399	3660	1943	$^{1}5838$	3568	1912	$^{1}5784$	3397	1852	$^{1}5679$
0.5	7488	3593	1889	$^{1}5643$	3508	1862	$^{1}5603$	3349	1811	$^{1}5525$
0.7	7629	3506	1824	$^{1}5427$	3430	1803	$^{1}5401$	3286	1761	$^{1}5351$
1.0	7764	3442	1782	$^{1}5301$	3372	1764	$^{1}5283$	3239	1728	$^{1}5248$
2.0	7927	3398	1759	$^{1}5244$	3331	1743	$^{1}5229$	3206	1710	$^{1}5201$

Similarly, the ceiling helios is

$$H_2 = \left(\frac{H_2}{H_{01}}\right) H_{01} + \left(\frac{H_2}{H_{02}}\right) H_{02} + \left(\frac{H_2}{H_{03}}\right) H_{03}, \quad (37)$$

and the floor helios is

$$H_3 = \left(\frac{H_3}{H_{01}}\right) H_{01} + \left(\frac{H_3}{H_{02}}\right) H_{02} + \left(\frac{H_3}{H_{03}}\right) H_{03}. \quad (38)$$

The average pharosage at table level is *

$$D_{AV} = \left(\frac{D_{AV}}{H_{01}}\right) H_{01} + \left(\frac{D_{AV}}{H_{02}}\right) H_{02} + \left(\frac{D_{AV}}{H_{03}}\right) H_{03}. \qquad (39)$$

TABLE X.

Ceiling Lighting, — C_2.

k_r	ρ_2=0.80 (Ceiling)				ρ_2=0.70			ρ_2=0.50		
	ρ_1=0.80	0.50	0.30	0.10	0.50	0.30	0.10	0.50	0.30	0.10
					ρ_3=0.10 (Floor)					
0	08750	03459	01754	15156	03422	01735	15101	03350	01698	14993
0.1	8637	3444	1755	15184	3399	1737	15142	3315	1702	15060
0.2	8539	3432	1756	15203	3382	1738	15170	3287	1704	15105
0.3	8453	3423	1757	15216	3369	1739	15189	3267	1705	15135
0.4	8378	3416	1757	15224	3360	1740	15201	3252	1706	15156
0.5	8315	3411	1758	15230	3352	1741	15210	3240	1707	15170
0.7	8213	3404	1758	15236	3342	1741	15220	3224	1708	15186
1.0	8111	3399	1758	15241	3335	1742	15225	3213	1709	15196
2.0	7986	3395	1759	15242	3329	1742	15228	3204	1709	15200
					ρ_3=0.30 (Floor)					
0	08238	03256	01651	14854	03133	01590	14670	02911	01476	14340
0.1	8195	3294	1686	14998	3184	1637	14872	2986	1547	14639
0.2	8158	3320	1709	15084	3222	1669	14996	3041	1596	14828
0.3	8126	3339	1724	15138	3249	1691	15074	3082	1629	14951
0.4	8100	3353	1734	15172	3269	1706	15125	3113	1653	15032
0.5	8078	3364	1741	15195	3284	1717	15158	3135	1669	15086
0.7	8043	3377	1750	15221	3304	1729	15196	3165	1689	15147
1.0	8008	3387	1755	15236	3318	1737	15218	3187	1702	15183
2.0	7969	3394	1759	15242	3328	1742	15228	3203	1709	15200

By use of equations (36) to (39), one can determine the important characteristics of any lighting design. These final values may be tabulated, if desired, for various types of luminaires, making use of the basic tables (Sections 5 to 7) and the known light distributions from the luminaires.

Instead of employing the quantities of equations (36) to equations (39), however, one may find practical advantage in the use of related quantities in the final tables that apply to specific luminaires. Define *interflectance* f of a room as the ratio of the pharos (lumens) on the

* Assuming that no direct light from the luminaires reaches the illuminated plane. With luminaires hung above desk level and giving a direct component of light, equation (39) must contain an added term F_3/S.

horizontal surface at table level to the pharos from the luminaires:

$$f = \frac{F_W}{F_D} = \frac{D_{AV}S}{F_D}, \tag{40}$$

where F_W = pharos (lumens) to a horizontal surface of area S at table level,

F_D = total pharos (lumens) from luminaires.

TABLE XI.

Ceiling Lighting, H_{1M}/H_{02}.

k_r	$\rho_2=0.80$ (Ceiling)				$\rho_2=0.70$			$\rho_2=0.50$		
	$\rho_1=0.80$	0.50	0.30	0.10	0.50	0.30	0.10	0.50	0.30	0.10
					$\rho_3=0.10$ (Floor)					
0	$^{0}4782$	$^{0}2989$	$^{0}1793$	$^{1}5978$	$^{0}2957$	$^{0}1774$	$^{1}5914$	$^{0}2895$	$^{0}1737$	$^{1}5789$
0.1	4855	2849	1643	$^{1}5269$	2813	1626	$^{1}5227$	2742	1593	$^{1}5143$
0.2	4891	2704	1506	$^{1}4683$	2665	1491	$^{1}4654$	2590	1461	$^{1}4595$
0.3	4895	2557	1382	$^{1}4187$	2517	1368	$^{1}4165$	2441	1342	$^{1}4122$
0.4	4873	2412	1268	$^{1}3759$	2372	1256	$^{1}3742$	2295	1232	$^{1}3710$
0.5	4825	2269	$^{0}1165$	$^{1}3385$	2230	$^{0}1154$	$^{1}3372$	2155	$^{0}1132$	$^{1}3346$
0.7	4673	1999	$^{0}9831$	$^{1}2762$	1962	$^{0}9738$	$^{1}2754$	1893	$^{0}9553$	$^{1}2736$
1.0	4351	$^{0}1639$	$^{0}7636$	$^{1}2055$	$^{0}1608$	$^{0}7562$	$^{1}2049$	$^{0}1549$	$^{0}7421$	$^{1}2037$
2.0	3080	$^{1}8220$	$^{1}3301$	$^{2}7871$	$^{1}8059$	$^{1}3271$	$^{2}7847$	$^{1}7758$	$^{1}3210$	$^{2}7808$
					$\rho_3=0.30$ (Floor)					
0	$^{0}6842$	$^{0}4276$	$^{0}2566$	$^{1}8553$	$^{0}4114$	$^{0}2468$	$^{1}8228$	$^{0}3823$	$^{0}2294$	$^{1}7647$
0.1	6658	3819	2172	$^{1}6883$	3692	2109	$^{1}6709$	3463	1994	$^{1}6387$
0.2	6465	3440	1878	$^{1}5740$	3339	1835	$^{1}5640$	3152	1754	$^{1}5451$
0.3	6264	3121	1648	$^{1}4901$	3037	1617	$^{1}4840$	2881	1558	$^{1}4723$
0.4	6061	2846	1462	$^{1}4253$	2775	1438	$^{1}4214$	2642	1393	$^{1}4137$
0.5	5855	2606	1307	$^{1}3732$	2544	1288	$^{1}3706$	2429	1253	$^{1}3654$
0.7	5446	2204	$^{0}1062$	$^{1}2941$	2155	$^{0}1049$	$^{1}2926$	2065	$^{0}1025$	$^{1}2899$
1.0	4852	$^{0}1739$	$^{0}7973$	$^{1}2124$	$^{0}1704$	$^{0}7890$	$^{1}2117$	$^{0}1637$	$^{0}7730$	$^{1}2103$
2.0	3199	$^{1}8325$	$^{1}3327$	$^{2}7912$	$^{1}8161$	$^{1}3292$	$^{2}7891$	$^{1}7855$	$^{1}3231$	$^{2}7849$

Also needed is

F_L = total pharos (lumens) from lamps.

Substitution of equation (39) in equation (40) gives

$$f = \rho_1\left(\frac{D_{AV}}{H_{01}}\right)\left(\frac{F_1}{F_D}\right)\frac{S}{S_1} + \rho_2\left(\frac{D_{AV}}{H_{02}}\right)\left(\frac{F_2}{F_D}\right) + \rho_3\left(\frac{D_{AV}}{H_{03}}\right)\left(\frac{F_3}{F_D}\right). \tag{41}$$

Here the flux ratios are obtained from the known light distributions of the luminaires, and

$$(F_1/F_D) + (F_2/F_D) + (F_3/F_D) = 1. \tag{42}$$

The *logance* (efficiency) of the luminaire is defined as

$$g = F_D/F_L \tag{43}$$

TABLE XII.

Ceiling Lighting, H_2/H_{02}.

k_r	ρ_2=0.80 (Ceiling)				ρ_2=0.70			ρ_2=0.50		
	ρ_1=0.80	0.50	0.30	0.10	0.50	0.30	0.10	0.50	0.30	0.10
					ρ_3=0.10 (Floor)					
0	1.0870	1.0870	1.0870	1.0870	1.0753	1.0753	1.0753	1.0526	1.0526	1.0526
0.1	1.1374	1.1040	1.0840	1.0655	1.0898	1.0727	1.0568	1.0626	1.0509	1.0399
0.2	1.1815	1.1172	1.0819	1.0512	1.1011	1.0709	1.0445	1.0702	1.0496	1.0314
0.3	1.2198	1.1273	1.0804	1.0416	1.1096	1.0697	1.0362	1.0759	1.0488	1.0256
0.4	1.2532	1.1350	1.0793	1.0352	1.1162	1.0687	1.0307	1.0803	1.0481	1.0217
0.5	1.2817	1.1410	1.0785	1.0308	1.1211	1.0680	1.0269	1.0836	1.0477	1.0191
0.7	1.3270	1.1487	1.0776	1.0258	1.1277	1.0672	1.0226	1.0880	1.0471	1.0160
1.0	1.3725	1.1545	1.0769	1.0229	1.1327	1.0667	1.0200	1.0913	1.0467	1.0142
2.0	1.4281	1.1593	1.0765	1.0215	1.1362	1.0664	1.0188	1.0936	1.0465	1.0134
					ρ_3=0.30 (Floor)					
0	1.3158	1.3158	1.3158	1.3158	1.2658	1.2658	1.2658	1.1765	1.1765	1.1765
0.1	1.3353	1.2741	1.2386	1.2066	1.2319	1.2028	1.1762	1.1553	1.1369	1.1199
0.2	1.3518	1.2441	1.1882	1.1415	1.2072	1.1609	1.1216	1.1397	1.1099	1.0840
0.3	1.3659	1.2223	1.1544	1.1008	1.1892	1.1326	1.0871	1.1282	1.0913	1.0607
0.4	1.3776	1.2062	1.1313	1.0745	1.1762	1.1130	1.0646	1.1196	1.0782	1.0453
0.5	1.3876	1.1943	1.1152	1.0572	1.1660	1.0994	1.0497	1.1132	1.0690	1.0350
0.7	1.4032	1.1789	1.0961	1.0379	1.1531	1.0831	1.0330	1.1048	1.0580	1.0234
1.0	1.4182	1.1675	1.0836	1.0267	1.1435	1.0724	1.0233	1.0985	1.0507	1.0165
2.0	1.4360	1.1600	1.0768	1.0216	1.1368	1.0665	1.0189	1.0940	1.0466	1.0134

and the *coefficient of utilization* is defined as

$$k_u = F_W/F_L = \frac{F_W}{F_D}\frac{F_D}{F_L} = fg. \tag{44}$$

Substitution of equations (40) and (43) in (44) gives

$$k_u = fg = \frac{D_{AV}S}{F_L}$$

or

$$D_{AV} = k_u \frac{F_L}{S}. \tag{45}$$

Equation (45) is the fundamental formula of lighting design. It was introduced by Harrison and Anderson [10] and has been widely used in lighting practice.[11] In the design of a lighting system, the interflectance

TABLE XIII.

Ceiling Lighting, H_3/H_{02}

k_r	$\rho_2=0.80$ (Ceiling)				$\rho_2=0.70$			$\rho_2=0.50$		
	$\rho_1=0.80$	0.50	0.30	0.10	0.50	0.30	0.10	0.50	0.30	0.10
					$\rho_3=0.10$ (Floor)					
0	$^{0}1087$	$^{0}1087$	$^{0}1087$	$^{0}1087$	$^{0}1075$	$^{0}1075$	$^{0}1075$	$^{0}1053$	$^{0}1053$	$^{0}1053$
0.1	$^{0}1019$	$^{1}9554$	$^{1}9173$	$^{1}8820$	$^{1}9432$	$^{1}9077$	$^{1}8748$	$^{1}9196$	$^{1}8892$	$^{1}8609$
0.2	$^{1}9519$	$^{1}8374$	$^{1}7745$	$^{1}7200$	$^{1}8253$	$^{1}7667$	$^{1}7154$	$^{1}8021$	$^{1}7514$	$^{1}7064$
0.3	$^{1}8863$	$^{1}7322$	$^{1}6543$	$^{1}5904$	$^{1}7207$	$^{1}6478$	$^{1}5873$	$^{1}6988$	$^{1}6352$	$^{1}5813$
0.4	$^{1}8231$	$^{1}6391$	$^{1}5530$	$^{1}4854$	$^{1}6285$	$^{1}5476$	$^{1}4833$	$^{1}6083$	$^{1}5371$	$^{1}4791$
0.5	$^{1}7626$	$^{1}5572$	$^{1}4675$	$^{1}3999$	$^{1}5475$	$^{1}4630$	$^{1}3983$	$^{1}5292$	$^{1}4542$	$^{1}3953$
0.7	$^{1}6509$	$^{1}4222$	$^{1}3343$	$^{1}2723$	$^{1}4145$	$^{1}3311$	$^{1}2714$	$^{1}3999$	$^{1}3249$	$^{1}2697$
1.0	$^{1}5078$	$^{1}2774$	$^{1}2023$	$^{1}1537$	$^{1}2721$	$^{1}2003$	$^{1}1533$	$^{1}2622$	$^{1}1966$	$^{1}1524$
2.0	$^{1}2127$	$^{2}6764$	$^{2}3794$	$^{2}2302$	$^{2}6633$	$^{2}3758$	$^{2}2296$	$^{2}6384$	$^{2}3688$	$^{2}2284$
					$\rho_3=0.30$ (Floor)					
0	$^{0}3947$	$^{0}3947$	$^{0}3947$	$^{0}3947$	$^{0}3797$	$^{0}3797$	$^{0}3797$	$^{0}3529$	$^{0}3529$	$^{0}3529$
0.1	3641	3337	3161	3001	3227	3069	2926	3026	2901	2786
0.2	3354	2841	2574	2352	2757	2515	2311	2602	2405	2234
0.3	3086	2430	2122	1878	2365	2081	1855	2243	2006	1810
0.4	2836	2087	1762	1518	2035	1734	1504	1937	1680	1476
0.5	2605	1798	1472	$^{0}1236$	1755	1451	$^{0}1227$	1675	1411	$^{0}1210$
0.7	2193	$^{0}1340$	$^{0}1037$	$^{1}8306$	$^{0}1311$	$^{0}1025$	$^{1}8267$	$^{0}1256$	$^{0}1001$	$^{1}8189$
1.0	$^{0}1688$	$^{1}8701$	$^{1}6213$	$^{1}4652$	$^{1}8523$	$^{1}6149$	$^{1}4636$	$^{1}8187$	$^{1}6024$	$^{1}4606$
2.0	$^{1}6955$	$^{1}2105$	$^{1}1159$	$^{2}6943$	$^{1}2063$	$^{1}1148$	$^{2}6924$	$^{1}1985$	$^{1}1127$	$^{2}6887$

is obtained from equation (41), the logance from equation (43), and the coefficient of utilization from equation (44). Equation (45) then allows the determination of the lamp pharos F_L required to obtain a given average pharosage D_{AV}.

[10] Ward Harrison and E. A. Anderson, *I. E. S. Trans.*, **11**, 67 (1916); **15**, 97 (1920).

[11] Parry Moon, "Scientific basis of illuminating engineering," New York, p. 348; 1936. Moon and Spencer, *Illum. Eng.*, (1946).

The designer must next calculate the helios of various surfaces in the room. We have shown [12] that the important visual factors are the ratios of these values of helios to the helios H_A to which the eyes are adapted. Thus there is an advantage in expressing helios of walls,

TABLE XIV.

Ceiling Lighting, D_{AV}/H_{02}.

k_r	$\rho_2=0.80$ (Ceiling)				$\rho_2=0.70$			$\rho_2=0.50$		
	$\rho_1=0.80$	0.50	0.30	0.10	0.50	0.30	0.10	0.50	0.30	0.10
				$\rho_3=0.10$ (Floor)						
0	1.0869	1.0869	1.0869	1.0869	1.0753	1.0753	1.0753	1.0526	1.0526	1.0526
0.1	1.0478	^{0}9906	^{0}9563	^{0}9245	^{0}9779	^{0}9464	^{0}9170	^{0}9534	^{0}9271	^{0}9024
0.2	1.0061	9002	8420	7914	8872	8335	7864	8623	8169	7765
0.3	^{0}9628	8160	7417	6802	8032	7344	6767	7788	7200	6698
0.4	9188	7384	6536	5864	7262	6472	5839	7028	6347	5788
0.5	8745	6674	5761	5065	6557	5705	5046	6338	5596	5007
0.7	7870	5435	4478	3792	5336	4435	3780	5148	4352	3755
1.0	6639	3973	^{0}3072	^{0}2467	3898	^{0}3042	^{0}2460	3756	^{0}2985	^{0}2446
2.0	3566	1384	^{1}8751	^{1}5933	1357	^{1}8668	^{1}5917	1306	^{1}8507	^{1}5885
				$\rho_3=0.30$ (Floor)						
0	1.3158	1.3158	1.3158	1.3158	1.2658	1.2658	1.2658	1.1765	1.1765	1.1765
0.1	1.2432	1.1505	1.0968	1.0483	1.1124	1.0650	1.0219	1.0432	1.0067	^{0}9730
0.2	1.1728	1.0131	^{0}9304	^{0}8607	^{0}9831	^{0}9090	^{0}8460	^{0}9281	^{0}8690	8178
0.3	1.1052	^{0}8968	7985	7205	8726	7834	7115	8278	7548	6947
0.4	1.0403	7968	6907	6102	7770	6796	6046	7397	6583	5941
0.5	^{0}9783	7100	6008	5207	6932	5923	5171	6618	5760	5106
0.7	8634	5672	4593	3845	5548	4539	3827	5316	4434	3803
1.0	7129	4081	^{0}3113	^{0}2481	3997	^{0}3081	^{0}2473	3840	^{0}3018	^{0}2472
2.0	3696	1396	^{1}8783	^{1}5938	1369	^{1}8699	^{1}5922	1318	^{1}8537	^{1}5890

ceiling, and floor in terms of the adaptation helios H_A. From equations (35) and (36)

$$H_1(u) = \rho_1 \left(\frac{H_1(u)}{H_{01}} \right) \frac{F_1}{S_1} + \rho_2 \left(\frac{H_1(u)}{H_{02}} \right) \frac{F_2}{S} + \rho_3 \left(\frac{H_1(u)}{H_{03}} \right) \frac{F_3}{S}.$$

If the eyes are adapted to the work, which has an incident pharosage

[12] Moon and Spencer, *J. Opt. Soc. Am.*, **35**, 233 (1945).

[13] Parry Moon, "Scientific basis," p. 588.

[14] A. A. Brainerd and M. Denning, *Illum. Eng.*, **36**, 1397 (1941); A. A. Brainerd and R. A. Massey, *Illum. Eng.*, **37**, 738 (1942); A. A. Brainerd, *Illum. Eng.*, **40**, 455 (1945); Parry Moon, *J. Opt. Soc. Am.*, **32**, 238 (1942).

D_{AV}, a reflectance ρ_A, and which fills the fovea, then

$$H_A = \rho_A D_{AV}. \tag{46}$$

Thus,

$$\frac{H_1(u)}{H_A} = \frac{\rho_1\left(\frac{H_1(u)}{H_{01}}\right)\left(\frac{F_1}{F_D}\right)\frac{S}{S_1} + \rho_2\left(\frac{H_1(u)}{H_{02}}\right)\left(\frac{F_2}{F_D}\right) + \rho_2\left(\frac{H_1(u)}{H_{03}}\right)\left(\frac{F_3}{F_D}\right)}{\rho_A\left[\rho_1\left(\frac{D_{AV}}{H_{01}}\right)\left(\frac{F_1}{F_D}\right)\frac{S}{S_1} + \rho_2\left(\frac{D_{AV}}{H_{02}}\right)\left(\frac{F_2}{F_D}\right) + \rho_3\left(\frac{D_{AV}}{H_{03}}\right)\left(\frac{F_3}{F_D}\right)\right]}$$

TABLE XV.

Floor Lighting, $B_3 = H_{1T}/H_{03}$.

k_r	ρ_2=0.80 (Ceiling)				ρ_2=0.70			ρ_2=0.50		
	ρ_1=0.80	0.50	0.30	0.10	0.50	0.30	0.10	0.50	0.30	0.10
					ρ_3=0.10 (Floor)					
0	^{0}7826	^{0}4891	^{0}2935	^{1}9782	^{0}4570	^{0}2742	^{1}9140	^{0}3947	^{0}2368	^{1}7895
0.1	7337	4299	2477	^{1}7938	4009	2315	^{1}7436	3449	2001	^{1}6456
0.2	6854	3768	2091	^{1}6479	3508	1951	^{1}6081	3008	1691	^{1}5298
0.3	6382	3295	1767	^{1}5313	3063	1652	^{1}4992	2621	1429	^{1}4359
0.4	5927	2876	1493	^{1}4368	2671	1397	^{1}4108	2281	1208	^{1}3593
0.5	5491	2507	^{0}1262	^{1}3599	2327	^{0}1181	^{1}3386	1985	^{0}1022	^{1}2965
0.7	4686	1900	^{1}9026	^{1}2451	1762	^{1}8443	^{1}2307	^{0}1500	^{1}7309	^{1}2023
1.0	3656	^{0}1248	^{1}5461	^{1}1383	^{0}1157	^{1}5108	^{1}1303	^{1}9833	^{1}4423	^{1}1143
2.0	1532	^{1}3044	^{1}1021	^{2}2072	^{1}2819	^{2}9583	^{2}1951	^{1}2394	^{2}8298	^{2}1713
					ρ_3=0.30 (Floor)					
0	^{0}9473	^{0}5921	^{0}3553	^{0}1184	^{0}5380	^{0}3228	^{0}1076	^{0}4412	^{0}2647	^{1}8823
0.1	8739	5006	2845	^{1}9004	4571	2609	^{1}8290	3782	2176	^{1}6964
0.2	8050	4261	2317	^{1}7056	3905	2138	^{1}6548	3253	1804	^{1}5584
0.3	7407	3646	1910	^{1}5635	3350	1769	^{1}5256	2804	1504	^{1}4525
0.4	6807	3131	1586	^{1}4553	2883	1474	^{1}4260	2422	1260	^{1}3691
0.5	6252	2696	^{0}1325	^{1}3707	2486	1233	^{1}3476	2094	^{0}1058	^{1}3024
0.7	5264	2011	^{1}9332	^{1}2492	1857	^{1}8709	^{1}2342	1570	^{1}7506	^{1}2047
1.0	4051	^{0}1305	^{1}5592	^{1}1396	1207	^{1}5226	^{1}1314	^{0}1023	^{1}4518	^{1}1151
2.0	1669	^{1}3156	^{1}1043	^{2}2083	^{1}2922	^{2}9760	^{2}1962	^{1}2481	^{2}8451	^{2}1722

or

$$\frac{H_1(u)}{D_{AV}} = \frac{\rho_A H_1(u)}{H_A} = \frac{1}{f}\left[\rho_1\left(\frac{H_1(u)}{H_{01}}\right)\left(\frac{F_1}{F_D}\right)\frac{S}{S_1} + \rho_2\left(\frac{H_1(u)}{H_{02}}\right)\left(\frac{F_2}{F_D}\right) + \rho_3\left(\frac{H_1(u)}{H_{03}}\right)\left(\frac{F_3}{F_D}\right)\right]. \tag{47}$$

This equation gives the helios ratio for a reflectance $\rho_A = 1.00$. For any other reflectance, the value $H_1(u)/D_{AV}$ must be divided by ρ_A to give the true helios ratio. Similarly,

$$\frac{H_2}{D_{AV}} = \frac{1}{f}\left[\rho_1\left(\frac{H_2}{H_{01}}\right)\left(\frac{F_1}{F_D}\right)\frac{S}{S_1} + \rho_2\left(\frac{H_2}{H_{02}}\right)\left(\frac{F_2}{F_D}\right) + \rho_3\left(\frac{H_2}{H_{03}}\right)\left(\frac{F_3}{F_D}\right)\right], \quad (48)$$

and

$$\frac{H_3}{D_{AV}} = \frac{1}{f}\left[\rho_1\left(\frac{H_3}{H_{01}}\right)\left(\frac{F_1}{F_D}\right)\frac{S}{S_1} + \rho_2\left(\frac{H_3}{H_{02}}\right)\left(\frac{F_2}{F_D}\right) + \rho_3\left(\frac{H_3}{H_{03}}\right)\left(\frac{F_3}{F_D}\right)\right]. \quad (49)$$

TABLE XVI.

Floor Lighting, C_3.

k_r	$\rho_2=0.80$ (Ceiling)				$\rho_2=0.70$			$\rho_2=0.50$		
	$\rho_1=0.80$	0.50	0.30	0.10	0.50	0.30	0.10	0.50	0.30	0.10
				$\rho_3=0.10$ (Floor)						
0	$^{0}1944$	$^{1}7686$	$^{1}3897$	$^{1}1146$	$^{0}1141$	$^{1}5783$	$^{1}1700$	$^{0}1861$	$^{1}9436$	$^{1}2774$
0.1	1823	$^{1}6756$	$^{1}3289$	$^{2}9297$	$^{0}1000$	$^{1}4882$	$^{1}1383$	1626	$^{1}7972$	$^{1}2269$
0.2	1703	$^{1}5921$	$^{1}2777$	$^{2}7589$	$^{1}8754$	$^{1}4124$	$^{1}1131$	1418	$^{1}6736$	$^{1}1862$
0.3	1586	$^{1}5177$	$^{1}2346$	$^{2}6223$	$^{1}7644$	$^{1}3485$	$^{2}9286$	1235	$^{1}5694$	$^{1}1532$
0.4	1472	$^{1}4519$	$^{1}1983$	$^{2}5117$	$^{1}6666$	$^{1}2945$	$^{2}7641$	$^{0}1075$	$^{1}4814$	$^{1}1263$
0.5	1364	$^{1}3940$	$^{1}1676$	$^{2}4215$	$^{1}5808$	$^{1}2490$	$^{2}6298$	$^{1}9356$	$^{1}4071$	$^{1}1042$
0.7	$^{0}1164$	$^{1}2986$	$^{1}1200$	$^{2}2870$	$^{1}4397$	$^{1}1781$	$^{2}4292$	$^{1}7070$	$^{1}2912$	$^{2}7107$
1.0	$^{1}9084$	$^{1}1962$	$^{2}7252$	$^{2}1620$	$^{1}2887$	$^{1}1078$	$^{2}2423$	$^{1}4635$	$^{1}1762$	$^{2}4015$
2.0	$^{1}3805$	$^{2}4783$	$^{2}1356$	$^{3}2426$	$^{2}7038$	$^{2}2021$	$^{3}3630$	$^{1}1129$	$^{2}3306$	$^{3}6017$
				$\rho_3=0.30$ (Floor)						
0	$^{0}2354$	$^{1}9304$	$^{1}4718$	$^{1}1387$	$^{0}1343$	$^{1}6808$	$^{1}2002$	$^{0}2080$	$^{0}1055$	$^{1}3100$
0.1	2171	$^{1}7866$	$^{1}3778$	$^{1}1055$	$^{0}1141$	$^{1}5502$	$^{1}1542$	1783	$^{1}8668$	$^{1}2447$
0.2	2000	$^{1}6695$	$^{1}3077$	$^{2}8264$	$^{1}9745$	$^{1}4509$	$^{1}1218$	1534	$^{1}7185$	$^{1}1962$
0.3	1840	$^{1}5728$	$^{1}2536$	$^{2}6599$	$^{1}8360$	$^{1}3732$	$^{2}9776$	1322	$^{1}5993$	$^{1}1590$
0.4	1691	$^{1}4920$	$^{1}2106$	$^{2}5333$	$^{1}7194$	$^{1}3109$	$^{2}7925$	$^{0}1142$	$^{1}5019$	$^{1}1297$
0.5	1553	$^{1}4237$	$^{1}1759$	$^{2}4342$	$^{1}6205$	$^{1}2601$	$^{2}6466$	$^{1}9872$	$^{1}4216$	$^{1}1063$
0.7	1308	$^{1}3159$	$^{1}1239$	$^{2}2918$	$^{1}4635$	$^{1}1837$	$^{2}4357$	$^{1}7402$	$^{1}2991$	$^{2}7194$
1.0	$^{0}1007$	$^{1}2051$	$^{2}7426$	$^{2}1635$	$^{1}3013$	$^{1}1102$	$^{2}2444$	$^{1}4824$	$^{1}1800$	$^{2}4046$
2.0	$^{1}4147$	$^{2}4959$	$^{2}1386$	$^{3}2440$	$^{2}7293$	$^{2}2059$	$^{3}3649$	$^{1}1170$	$^{2}3367$	$^{3}6050$

9. SUMMARY.

The fundamental equation in the design of lighting systems for rooms is

$$D_{AV} = k_u \frac{F_L}{S} \quad (45)$$

where D_{AV} = average pharosage (lumens m^{-2}) on horizontal surface at table level,
F_L = total pharos (lumens) from lamps,
S = floor area (m^2).

The *coefficient of utilization* is

$$k_u = fg \tag{44}$$

TABLE XVII.

Floor Lighting, H_{1M}/H_{03}.

k_r	$\rho_2=0.80$ (Ceiling)				$\rho_2=0.70$			$\rho_2=0.50$		
	$\rho_1=0.80$	0.50	0.30	0.10	0.50	0.30	0.10	0.50	0.30	0.10
					$\rho_3=0.10$ (Floor)					
0	$^{0}7826$	$^{0}4891$	$^{0}2935$	$^{1}9782$	$^{0}4570$	$^{0}2742$	$^{1}9140$	$^{0}3947$	$^{0}2368$	$^{1}7895$
0.1	7426	4358	2513	$^{1}8062$	4089	2364	$^{1}7601$	3572	2075	$^{1}6701$
0.2	7033	3890	2167	$^{1}6741$	3667	2048	$^{1}6406$	3239	1828	$^{1}5749$
0.3	6653	3480	1882	$^{1}5709$	3296	1793	$^{1}5464$	2944	1619	$^{1}4979$
0.4	6287	3122	1645	$^{1}4886$	2962	1576	$^{1}4704$	2681	1441	$^{1}4346$
0.5	5936	2808	1447	$^{1}4219$	2684	1393	$^{1}4084$	2448	1288	$^{1}3817$
0.7	5288	2291	1136	$^{1}3216$	2209	$^{0}1104$	$^{1}3141$	2052	$^{0}1040$	$^{1}2993$
1.0	4448	$^{0}1724$	$^{1}8166$	$^{1}2232$	$^{0}1680$	$^{1}8014$	$^{1}2200$	$^{0}1595$	$^{1}7716$	$^{1}2138$
2.0	2574	$^{1}7556$	$^{1}3165$	$^{2}7854$	$^{1}7502$	$^{1}3163$	$^{2}7836$	$^{1}7399$	$^{1}3139$	$^{2}7800$
					$\rho_3=0.30$ (Floor)					
0	$^{0}9473$	$^{0}5921$	$^{0}3553$	$^{0}1184$	$^{0}5380$	$^{0}3228$	$^{0}1076$	$^{0}4412$	$^{0}2647$	$^{1}8823$
0.1	8845	5074	2886	$^{1}9145$	4663	2664	$^{1}8474$	3918	2256	$^{1}7228$
0.2	8262	4398	2401	$^{1}7340$	4082	2244	$^{1}6899$	3503	1950	$^{1}6059$
0.3	7722	3850	2034	$^{1}6055$	3604	1920	$^{1}5752$	3150	1704	$^{1}5168$
0.4	7221	3398	1748	$^{1}5092$	3205	1663	$^{1}4879$	2847	1502	$^{1}4464$
0.5	6759	3020	1518	$^{1}4346$	2867	1459	$^{1}4193$	2583	1334	$^{1}3894$
0.7	5940	2425	$^{0}1175$	$^{1}3270$	2328	$^{0}1138$	$^{1}3189$	2148	$^{0}1069$	$^{1}3029$
1.0	4929	$^{0}1803$	$^{1}8362$	$^{1}2252$	$^{0}1753$	$^{1}8198$	$^{1}2219$	$^{0}1660$	$^{1}7881$	$^{1}2154$
2.0	2805	$^{1}7833$	$^{1}3234$	$^{2}7896$	$^{1}7776$	$^{1}3222$	$^{2}7878$	$^{1}7668$	$^{1}3197$	$^{2}7841$

where the *interflectance f* depends only on the room shape and reflectances, while the *logance g* depends only on the luminaire. The room shape is specified by the *room index* k_r, which according to the integral-equation theory of interflections is

$$k_r = \frac{ph}{4S} \tag{15}$$

where p = perimeter (m) of the horizontal cross section of the room,
h = height of room.

Equation (16) applies to a room of *any shape*, provided a flat ceiling is used. For the special case of a rectangular room,

$$k_r = \frac{h(l+w)}{2lw}. \tag{15}$$

The integral equation for interflections of light in rooms is expressed

TABLE XVIII.

Floor Lighting, H_2/H_{02}.

k_r	$\rho_2=0.80$ (Ceiling)				$\rho_2=0.70$			$\rho_2=0.50$		
	$\rho_1=0.80$	0.50	0.30	0.10	0.50	0.30	0.10	0.50	0.30	0.10
					$\rho_3=0.10$ (Floor)					
0	$^{0}8696$	$^{0}8696$	$^{0}8696$	$^{0}8696$	$^{0}7527$	$^{0}7527$	$^{0}7527$	$^{0}5263$	$^{0}5263$	$^{0}5263$
0.1	8152	7644	7339	7056	6602	6354	6124	4598	4446	4304
0.2	7615	6699	6197	5760	5777	5367	5008	4011	3757	3532
0.3	7091	5858	5235	4723	5045	4535	4111	3494	3176	2906
0.4	6585	5113	4425	3883	4400	3833	3383	3042	2685	2395
0.5	6101	4457	3740	3199	3833	3241	2788	2646	2271	1976
0.7	5207	3378	2675	2179	2902	2318	1900	2000	$^{0}1624$	$^{0}1349$
1.0	4063	$^{0}2219$	$^{0}1618$	$^{0}1230$	$^{0}1905$	$^{0}1402$	$^{0}1073$	$^{0}1311$	$^{1}9829$	$^{1}7619$
2.0	1702	$^{1}5412$	$^{1}3035$	$^{1}1842$	$^{1}4643$	$^{1}2630$	$^{1}1607$	$^{1}3192$	$^{1}1844$	$^{1}1142$
					$\rho_3=0.30$ (Floor)					
0	1.0526	1.0526	1.0526	1.0526	$^{0}8861$	$^{0}8861$	$^{0}8861$	$^{0}5882$	$^{0}5882$	$^{0}5882$
0.1	$^{0}9710$	$^{0}8900$	$^{0}8429$	$^{0}8004$	7529	7161	6827	5043	4835	4643
0.2	8945	7575	6865	6272	6432	5869	5393	4337	4008	3723
0.3	8230	6481	5658	5009	5518	4857	4328	3739	3343	3017
0.4	7564	5566	4700	4047	4748	4046	3509	3229	2799	2461
0.5	6947	4793	3925	3295	4095	3385	2863	2792	2351	2016
0.7	5849	3574	2765	2215	3059	2391	1929	2094	1668	$^{0}1365$
1.0	4501	$^{0}2320$	$^{0}1657$	$^{0}1241$	$^{0}1989$	$^{0}1435$	$^{0}1082$	$^{0}1365$	$^{0}1004$	$^{1}7676$
2.0	1855	$^{1}5610$	$^{1}3091$	$^{1}1851$	$^{1}4813$	$^{1}2679$	$^{1}1616$	$^{1}3308$	$^{1}1878$	$^{1}1148$

in terms of the room index and is solved to give expressions for D_{AV}/H_0 for three types of lighting:

(a) Wall lighting,
(b) Ceiling lighting,
(c) Floor lighting.

These results are computed and tabulated (Tables VIII, XIV, XX). Any more complicated system of lighting may be considered as a

combination of (a), (b), and (c). Then the interflectance is

$$f = \rho_1 \left(\frac{D_{AV}}{H_{01}}\right)\left(\frac{F_1}{F_D}\right)\frac{S}{S_1} + \rho_2 \left(\frac{D_{AV}}{H_{02}}\right)\left(\frac{F_2}{F_D}\right) + \rho_3 \left(\frac{D_{AV}}{H_{03}}\right)\left(\frac{F_3}{F_D}\right). \quad (41)$$

Here the values of (D_{AV}/H_0) are obtained from Tables VIII, XIV, and XX, while (F_i/F_D) represents the fractional part of the light from the luminaires that is incident on the walls, ceiling, or floor. By use of the tables and equation (41), the interflectance can be obtained for any

TABLE XIX.

Floor Lighting, H_3/H_{03}.

k_r	$\rho_2=0.80$ (Ceiling)				$\rho_2=0.70$			$\rho_2=0.50$		
	$\rho_1=0.80$	0.50	0.30	0.10	0.50	0.30	0.10	0.50	0.30	0.10
					$\rho_3=0.10$ (Floor)					
0	1.0870	1.0870	1.0870	1.0870	1.0753	1.0753	1.0753	1.0526	1.0526	1.0526
0.1	1.0803	1.0705	1.0646	1.0593	1.0615	1.0563	1.0516	1.0442	1.0402	1.0365
0.2	1.0743	1.0579	1.0487	1.0408	1.0509	1.0427	1.0357	1.0377	1.0313	1.0256
0.3	1.0692	1.0480	1.0374	1.0286	1.0428	1.0331	1.0251	1.0327	1.0249	1.0183
0.4	1.0647	1.0406	1.0293	1.0203	1.0366	1.0263	1.0180	1.0289	1.0204	1.0134
0.5	1.0607	1.0350	1.0235	1.0147	1.0320	1.0213	1.0131	1.0262	1.0171	1.0100
0.7	1.0548	1.0275	1.0164	1.0083	1.0257	1.0153	1.0075	1.0224	1.0132	1.0061
1.0	1.0487	1.0218	1.0117	1.0045	1.0210	1.0113	1.0042	1.0196	1.0105	1.0037
2.0	1.0413	1.0177	1.0091	1.0027	1.0177	1.0091	1.0027	1.0177	1.0091	1.0027
					$\rho_3=0.30$ (Floor)					
0	1.3158	1.3158	1.3158	1.3158	1.2658	1.2658	1.2658	1.1765	1.1765	1.1765
0.1	1.2867	1.2464	1.2228	1.2016	1.2104	1.1904	1.1723	1.1453	1.1311	1.1180
0.2	1.2619	1.1961	1.1619	1.1334	1.1700	1.1403	1.1153	1.1222	1.1000	1.0809
0.3	1.2409	1.1596	1.1212	1.0908	1.1405	1.1065	1.0793	1.1051	1.0786	1.0569
0.4	1.2229	1.1328	1.0933	1.0634	1.1187	1.0831	1.0558	1.0923	1.0637	1.0410
0.5	1.2080	1.1130	1.0740	1.0452	1.1025	1.0669	1.0402	1.0829	1.0532	1.0303
0.7	1.1848	1.0873	1.0508	1.0251	1.0814	1.0473	1.0228	1.0704	1.0405	1.0183
1.0	1.1620	1.0683	1.0359	1.0134	1.0658	1.0346	1.0127	1.0612	1.0322	1.0112
2.0	1.1349	1.0551	1.0277	1.0081	1.0549	1.0277	1.0081	1.0547	1.0276	1.0080

room with flat ceiling. Equations (44) and (45) then allow the design of the lighting system to provide any desired D_{AV}.

Next it is necessary to find if the helios distribution for the proposed lighting system is satisfactory. This question is answered by reference to the tables of ceiling helios, wall helios, and floor helios. All these values were calculated by means of the equations resulting from the integral equation for interflections.

It is believed that this method gives a new and powerful tool for the design of lighting systems and the solution of problems of interior illumination and decoration. For the first time, it becomes possible to predetermine not only the pharosage on a horizontal surface but the pharosage on any surface, as well as the entire helios distribution in the room.

TABLE XX.

Floor Lighting, D_{AV}/H_{03}.

k_r	$\rho_2=0.80$ (Ceiling)				$\rho_2=0.70$			$\rho_2=0.50$		
	$\rho_1=0.80$	0.50	0.30	0.10	0.50	0.30	0.10	0.50	0.30	0.10
					$\rho_3=0.10$ (Floor)					
0	$^{0}8696$	$^{0}8696$	$^{0}8696$	$^{0}8696$	$^{0}7527$	$^{0}7527$	$^{0}7527$	$^{0}5263$	$^{0}5263$	$^{0}5263$
0.1	8057	7183	6665	6185	6249	5797	5376	4451	4114	3797
0.2	7455	5964	5148	4440	5221	4502	3874	3798	3250	2763
0.3	6914	4984	4013	3216	4396	3532	2816	3273	2599	2029
0.4	6424	4197	3160	2348	3733	2802	2065	2850	2107	1505
0.5	5981	3567	2518	$^{0}1730$	3202	2252	$^{0}1528$	2507	1733	$^{0}1129$
0.7	5225	2654	1665	$^{1}9688$	2429	$^{0}1517$	$^{1}8659$	2002	$^{0}1229$	$^{1}6620$
1.0	4355	$^{0}1844$	$^{0}1004$	$^{1}4513$	$^{0}1736$	$^{1}9232$	$^{1}4135$	$^{0}1532$	$^{1}8229$	$^{1}3385$
2.0	2799	$^{1}9140$	$^{1}4108$	$^{1}1126$	$^{1}9048$	$^{1}4075$	$^{1}1112$	$^{1}8875$	$^{1}4011$	$^{1}1085$
					$\rho_3=0.30$ (Floor)					
0	1.0526	1.0526	1.0526	1.0526	$^{0}8861$	$^{0}8861$	$^{0}8861$	$^{0}5882$	$^{0}5882$	$^{0}5882$
0.1	$^{0}9596$	$^{0}8364$	$^{0}7655$	$^{0}7016$	7126	6533	5993	4882	4473	4096
0.2	8756	6743	5704	4835	5813	4924	4172	4107	3467	2912
0.3	8025	5514	4337	3410	4807	3783	2965	3502	2735	2106
0.4	7379	4569	3357	2447	4028	2958	2141	3025	2197	1546
0.5	6811	3836	2643	$^{0}1782$	3420	2352	$^{0}1569$	2646	1795	$^{0}1152$
0.7	5869	2809	1722	$^{1}9850$	2561	$^{0}1565$	$^{1}8791$	2096	$^{0}1262$	$^{1}6701$
1.0	4826	$^{0}1928$	$^{0}1028$	$^{1}4553$	$^{0}1812$	$^{1}9445$	$^{1}4170$	$^{0}1594$	$^{1}8405$	$^{1}3410$
2.0	3051	$^{1}9475$	$^{1}4184$	$^{1}1132$	$^{1}9379$	$^{1}4150$	$^{1}1118$	$^{1}9198$	$^{1}4085$	$^{1}1091$

APPENDIX.

A summary of the symbols is given below. Note that subscript 1 is employed throughout to refer to the walls, subscript 2 to the ceiling, and subscript 3 to the floor. A subscript 0 refers to a condition obtained without interflections.

a = exponent in eq. (9)

$$= \frac{\rho w}{2S}.$$

$A = a/2$, constant in eq. (9).

B_1, B_2, B_3 = coefficients of cosh ks in eqs. (11), (18), (23), (28).

C_1, C_2, C_3 = coefficients of sinh ks in eqs. (11), (18), (23), (28).

D = pharosage (lumens m^{-2}).

D_{AV} = average pharosage incident on the top of an imaginary plane surface at table level.

E = constant defined by eq. (12).

f = interflectance
$= \frac{F_W}{F_D}$, eq. (40).
F_1 = pharos to walls (lumen).
F_2 = pharos to ceiling (lumen).
F_3 = pharos to floor (lumen).
F_D = total pharos from luminaires (lumen).
F_L = total pharos from lamps (lumen).
F_W = total pharos incident on the top of an imaginary plane surface at table level.
g = logance
$= F_D/F_L$, eq. (43).
h = ceiling height (m).
h' = distance from ceiling to table level (m).
H_1 = helios of walls, including interflections (blondel).
H_2 = helios of ceiling, including interflections (blondel).
H_3 = helios of floor, including interflections (blondel).
H_{01} = helios of walls without interflections (blondel).
H_{02} = helios of ceiling without interflections (blondel).
H_{03} = helios of floor without interflections (blondel).
H_{1M} = helios at middle of wall (blondel).
H_{1T} = helios at top of wall (blondel).
H_A = helios to which the fovea is adapted (blondel).
$k = a(1 - \rho_1)^{1/2}$.
k_r = room index, eqs. (14), (15).
k_u = coefficient of utilization = F_W/F_L.
$K_{11}(s, t)$ = kernel of the integral equation, eq. (1).
$K_{12}(s)$, $K_{13}(s)$, $K_{21}(s)$, $K_{31}(s)$, K_{32}, K_{23} = quantities defined in Section 4.
l = length of room (m).
p = perimeter of room (m).
$s = u/w$.
S = area of floor (m^2).
S_1 = total area of walls (m^2).
$t = v/w$.
u = distance from ceiling to point P, Figs. 1 and 2.
v = distance from ceiling to luminous band, Figs. 1 and 2.
w = width of rectangular room (m), or a dimension of a non-rectangular room.
ρ_1 = average wall reflectance.
ρ_2 = average ceiling reflectance.
ρ_3 = average floor reflectance.
ρ_A = reflectance of surface to which the eye is adapted.

New Permanent-Magnet Materials.—Five new permanent-magnet materials, known as cunico, cunife, vectolite, alnico 12 and silmanal, have been announced by the General Electric Company. These new materials have many applications in the manufacture of aircraft instruments, as well as other industrial uses where small, lightweight magnets formed in intricate shapes are required.

Cunico is an alloy of copper, nickel and cobalt. Cunico magnets are made from rod, strip or wire stock, and are furnished only in their final shapes because they are age-hardened in the manufacturing process. Cunico is malleable, ductile and machinable, permitting the manufacture of small magnet screws from this material by ordinary screw machine methods, and the punching of intricate magnet shapes.

Cunife is a copper-nickel-iron alloy which has all the physical properties of cunico, differing in that it has directional properties and must be magnetized only along the direction in which the material has been cold-worked, to obtain highest magnetic quality. Cunife magnets are made from wire stock in round, square or rectangular form. In addition, cunife wire can be flattened to make thin, narrow shapes. A wide variety of magnet designs can be obtained by forming, drawing, punching or machining cunife to shape.

Vectolite is the first non-metallic, non-conducting magnet material ever made. It is a hardened, sintered combination of iron rust and cobalt oxide mixed when still in powder form. Vectolite magnets are extremely light, being non-metallic, and their non-conductive properties prevent electrical losses caused by conduction of current. In addition, they have a high coercive force, or resistance to demagnetizing forces.

Alnico 12 is made up of aluminum, nickel, cobalt, iron and titanium. Magnets made from this material can be used in such applications as in tachometer generators and electronic devices. Alnico 12 magnets are cast, and must be ground to shape.

Silmanal has a high intrinsic coercive force, which makes magnets made from this material useful in instruments where service in strong electrical fields is required. Silmanal is ductile and malleable and can be punched, machined, rolled, or ground. Because of its ductility, silmanal can be made in rod, strip or wire form.

R. H. O.

NOTES FROM THE NATIONAL BUREAU OF STANDARDS.*

DESCRIPTION AND ANALYSIS OF THE FIRST SPECTRUM OF URANIUM.

Various problems concerning uranium require for their interpretation a knowledge of the energy states to which the neutral and ionized atoms of the element may be excited. These can be established only by an analysis of the spectra emitted by the atoms. Although some work had been done prior to 1942 in recording the uranium spectra, no differentiation had been set up between the spectra radiated by the atoms according to their stage of ionization; no resolutions of the lines in magnetic fields had been reported; and no reliable descriptions of the lines, such as intensities, reversibilities, and pressure shifts, had been published. It was to remove these deficiencies in our knowledge and to supply the data for setting up the system of energy levels of the atoms that the Bureau's Spectroscopy Section began an investigation of the uranium spectra late in 1942.

C. C. Kiess, C. J. Humphreys, and D. D. Laun are co-authors of a preliminary report on the description and analysis of the first spectrum of uranium, published in the *Journal of Research* for July (RP 1729).

As a result of the observations of the light emitted by uranium electrodes under arc and spark excitation the wavelengths, wavenumbers, and estimated intensities have been complied for more than 9000 lines in the spectrum of the neutral atom for the region from 11000A in the infrared to 2900A in the ultraviolet. About 2000 of these lines have been classified as combinations between 18 low and metastable odd energy levels with about 280 high even levels. Well-resolved Zeeman patterns give g-values for several low levels that identify them as components of ^{5}L, ^{5}K, ^{7}M, and ^{7}K terms arising in the electron configurations $f^3\,ds^2$ and $f^3\,d^2s$.

The spectrum of uranium is interpreted as that of a rare-earth element analogous to neodymium, uranium being the third member of a second group of rare earths beginning with thorium. From the fact that uranium is easily ionized in electric arcs and magnetic fields, and also that the short-wave limit of the observed spectrum does not extend below 2900A, it is concluded that the ionization potential of neutral uranium atoms is approximately 4 volts.

THERMAL CONDUCTIVITY OF INSULATING MATERIALS.

A program designed to provide more accurate and dependable values of thermal conductivity of insulating materials has been announced by

* Communicated by the Director.

the American Society of Heating and Ventilating Engineers, in cooperation with the National Bureau of Standards. Laboratories which meet the Society's requirements will be certified for testing thermal insulating materials under this program. The Bureau is cooperating by making check tests for accrediting equipment to be used by the participating laboratories.

Present tables of conductivity values of insulating materials are, in general, compilations of data obtained over a period of years by a number of different laboratories. In an effort to provide the information on a more uniform basis, and to take into account changes and improvements in currently available insulating materials, the ASHVE has initiated this program in which all data will be obtained in accordance with the American Society for Testing Materials "Standard Method of Test for Thermal Conductivity of Materials by Means of the Guarded Hot Plate" (ASTM Designation: C177-45). This method has recently been adopted by the ASHVE as their standard method for hot plate thermal conductivity measurements. All laboratories having equipment conforming to the requirements of this method are invited to participate.

According to this plan the ASHVE Research Laboratory will send a selected specimen of insulating material to a participating laboratory for test. After test, the specimen will be returned, together with test results, to the ASHVE Research Laboratory. The same specimen will then be sent to the Bureau, where it will again be tested. The basis of accrediting laboratories will be their ability to check within acceptable tolerances the values obtained at the Bureau.

A list of accredited laboratories will be made available by the ASHVE, so that manufacturers of thermal insulating materials may have their products tested under this plan. The test data may then be submitted to the ASHVE laboratory for review and compilation in the Heating, Ventilating, and Air Conditioning Guide. It is anticipated that beginning with the 1949 Guide, all data therein on the thermal conductivity of building insulations will have been obtained in accordance with this program.

Inquiries concerning the program should be addressed to the ASHVE Research Laboratory, 7216 Euclid Avenue, Cleveland 3, Ohio.

WATER-VAPOR PERMEABILITY OF MOISTURE BARRIERS.

An important test of performance of wrapping papers and caselining materials is the rate at which moisture will pass through the material under stated conditions. In connection with the work on standards for caseliners reported elsewhere in this Bulletin, equipment has been developed to facilitate testing of such moisture barriers under tropical

conditions. The apparatus is described in detail by F. T. Carson and V. Worthington of the Bureau's Paper Section, in the March 1946 issue of *The Paper Industry and Paper World*.

The test is usually made by sealing a specimen of known area in the mouth of a shallow dishlike cell containing a desiccant, such as anhydrous calcium chloride or magnesium perchlorate, and exposing the cell at 100°F. in an atmosphere of 90 per cent. relative humidity. The cell is weighed periodically to determine the rate at which moisture passes through the specimen to be absorbed by the desiccant. The cells are commonly exposed in a conditioning cabient, but are removed at intervals for weighing. This procedure results in a break in the continuity of the testing conditions, a temporary change in the conditions in the cabinet, condensation of moisture on the specimen when it is put back in the hot, moist atmosphere, and an uncertainty about the actual time of exposure to the stated conditions.

The chief feature of the apparatus developed at the National Bureau of Standards is a means of weighing the permeability cells at any time without taking them out of the cabinet or disturbing the conditioning atmosphere. A number of the prepared cells are hung by hooks from a notched, rotary disk in the top of the cabinet. By means of specially designed mechanism, each cell in turn is picked up by a weighing hook and suspended from the arm of a chemical balance standing on top of the cabinet. After a cell has been weighed a simple mechanical operation replaces it with the next one selected for weighing.

The testing equipment includes also an improved form of conditioning cabinet, provided with an air bath that maintains a constant, uniform temperature throughout. The atmosphere of the cabinet is circulated by a fan and is passed over a saturated salt solution suitably chosen to provide the desired relative humidity. Means are provided for putting in fresh specimens and removing others from the cabinet without disturbing the conditions in the cabinet and without condensing moisture on the added specimens. A special design of permeability cell is used with the apparatus.

DETERMINATION OF CURVATURE BY AN OSCULOMETER.

An important method of treating experimental observations is to plot a curve showing the relationship between two observed quantities; for instance, a curve showing the barometric pressure at a given place as a function of time, or one showing the gasoline consumption of an automobile as a function of speed. In studying such curves it is sometimes useful to be able to measure the curvature at points along the curve.

In a paper prepared for publication in the Review of Scientific Instruments, Harvey L. Curtis describes an instrument known as an

osculometer which he has developed for this purpose. In its simplest form, the osculometer consists of a series of arcs of circles drawn on a transparent plate, in which succeeding arcs of known curvature have a definite increase in curvature from one arc to the next. In use, the curvature is determined by selecting the arc which has the same curvature as the curve at the point under consideration. The instrument has been used with success to obtain the acceleration of a body from a displacement-time curve of its motion.

STANDARD BUFFER SOLUTIONS.

For the purpose of calibrating pH assemblies such as the hydrogen-calomel and the glass-calomel type, it is necessary to have on hand a number of salts of adequate purity from which buffer solutions of known pH can be prepared.

In the Symposium on pH sponsored in June 1946 by the American Society for Testing Materials in Buffalo, New York, George G. Manov presented details concerning the purification of potassium dihydrogen phosphate, disodium hydrogen phosphate (used together for the preparation of phosphate buffers), and borax (sodium tetraborate decahydrate). Each salt was purified by fractional recrystallization from water until it was evident that no error greater than 0.002 pH unit could be ascribed to residual impurities in the finished product. These salts were recently made available for distribution as NBS Standard Samples, 186-I, 186-II, and 187, respectively.

Analyses of commercial products were given showing the range of pH values for 0.1-m solutions of the ACS or reagent-grade chemicals.

Cells without liquid junction were used to measure the electromotive force of hydrogen and silver-silver-chloride electrodes immersed in a solution of the buffer to which had been added a known amount of sodium chloride. The pH values of the buffer mixtures were calculated from the data. The assumptions made in the method used at the Bureau in the calculation of pH were discussed, and a comparison was given of the pH values for the same buffers on the basis of cells with and without liquid junction.

Buffers covering the pH range from 1 to 9 were suggested for use in the calibration of pH equipment, together with data regarding their stabilities in aqueous solution over a long period of time.

PERMEABILITY OF BRICK-MORTAR ASSEMBLAGES.

Recent work on permeability of brick-mortar assemblages was described in a paper presented by John W. McBurney at the June meeting of the American Society for Testing Materials. For the purpose of this

investigation, small panels, two-brick long by three-brick high, and one-brick (3¾ inches) in thickness were built. Ten different kinds of brick were used. The average initial rates of absorption of these bricks ranged from 0.5 to 127 grams per minute, and the other properties, such as absorption, porosity, air permeability, and strength, also covered a considerable range. Mortars were of three types, corresponding to the A, B, and C classifications of the American Standard Building Code requirements for Masonry. Good workmanship (all joints filled) was used throughout. In addition to the building of panels from these ten kinds of brick with three kinds of mortar, panels were built with the same kinds of brick wetted before laying with mortar A.

Under the conditions of these tests, only panels built from bricks with initial rates of absorption (penetrabilities) of less than 10 grams per minute, and laid with mortar A, characteristically gave excellent results, i.e., no dampness or leakage up to 48 hours. All panels built from bricks with the higher initial rates of absorption (26 to 127 grams per minute) characteristically gave dampness or leakage (0 to over 5 liters per hour), irrespective of the mortar used. In general, the rate of leakage tended to increase with increase in initial rate of absorption of the bricks.

Prewetting of bricks that had medium to high initial rates of absorption greatly improved results of the permeability tests in comparison with the same bricks laid dry and with mortar A used throughout.

Comparisons of permeability results on panels retested after 1 and 2½ years' exposure, with initial tests made at 35 days, indicate, in general, that leakage decreased. Exceptions were provided by panels built from certain high absorption and low strength bricks that tended to disintegrate on exposure outdoors. The bricks that performed poorly had saturation coefficients exceeding 0.80.

By means of permeability tests, using water solutions of dyes of contrasting colors applied for long and short times, the nature, extent, and location of through channels in permeable masonry specimens can be observed when the specimens are dissected. For the panels so examined, the most probable location of a single through channel is at the interface between the mortar and cut surface of a half brick.

The work was carried out by John W. McBurney, M. Arnold Copeland, and Russell H. Brink of the Masonry Construction Section.

Cotton Tire Cords Stand Up.—Research on tire-cord materials was started early in the war to provide the armed forces with the most dependable tires that could be produced, says the U. S. Department of Agriculture. The Southern Regional Research Laboratory of the Bureau of Agricultural and Industrial Chemistry at New Orleans was one of the research groups working to improve cotton cord. To develop a cord that would be satisfactory for use with synthetic rubber, laboratory scientists first investigated what is known as low-gage cotton cord, which they believed would outwear and outrun the larger or high-gage cord that has been in use for many years. Then they selected certain commercially available varieties of cotton from which they believed better cord could be made. The first experimental tires were for trucks, made with 90 per cent. synthetic and 10 per cent. natural rubber. They were officially tested at the Army ordnance tire-testing grounds at San Antonio, Tex.

All tires tested provided satisfactory results, but those made with cord from improved varieties gave higher mileage and had better resistance to rocks and other obstacles than those of standard cotton cord. Tires made with cotton of the Stoneville variety were roughly 20 per cent. better than the standard cord; those of SxP cotton, 75 per cent. better; and those made from Wilds cotton, 132 per cent. better. These tests were made in 1943. Tests in the summer and fall of 1944 showed that rayon and the improved cotton cord performed better in light-truck tires than standard cotton cord.

In passenger car tests, in which no rayon cords were tested, both standard and improved cotton cords gave entirely satisfactory performance. With one recapping, both standard and improved cords ran a total of 68,000 miles and were still in good enough condition for another recapping. Thus the standard as well as the improved cotton cord was shown to be entirely adequate for high-speed passenger-car driving.

R. H. O.

Investment on Future Benefits.—"Many of our leading industries have long looked upon research as an investment in the future," comments P. V. Cardon in the closing paragraphs of his report as Administrator of Agricultural Research. "Between 1920 and 1940 expenditures for industrial research increased seven times. During the war the entire Nation came to look upon research as a practical necessity. In fact, obstacles that threatened to slow up progress of the war yielded to scientific research with such regularity that the public has developed a wholesome respect for science and the scientific approach.

"Military necessity was the only inducement needed to divert men and equipment from one research job to another. It was the only inducement needed to mobilize the best brains in the service of their country and focus their efforts upon a common undertaking. Results of war research are well known to all. We must realize that research is as good for us in peace as it is in war. There is ample evidence to support this belief.

"We now face the job of directing our research efforts into channels that will yield the greatest benefits to all people everywhere."

R. H. O.

THE FRANKLIN INSTITUTE.

LIBRARY NOTES.

The Committee on Library desires to add to the collections any technical works that members would wish to contribute. Contributions will be gratefully acknowledged and placed in the library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat Service. Photostat prints of any material in the collections can be supplied on request. Orders received in the morning are filled the same day. The average cost for a print 9 × 14 inches is thirty-five cents.

The library and reading room are open on Mondays, Tuesdays, Fridays and Saturdays from nine o'clock A.M. until five o'clock P.M., Wednesdays and Thursdays from two until ten o'clock P.M.

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NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION.

A Microdensitometer for Quantitative Determination of Relative Densities of Photographic Negatives of Tissue Cells.—T. ENNS. A densitometer has been constructed for examination of photomicrographic negatives of individual cells. When monochromatic light is passed through thin sections of tissue some areas absorb more light than do others. For this reason when photomicrographs are taken of such tissue the film density varies in different areas inversely with the amount of light absorbed by the corresponding areas of the tissue section. Cell nuclei of biological material absorb light in varying degree depending on the wave length used. Visual observation of the film or of photographs made from such film is not sufficiently precise for accurately recording the differences of absorption at various wave lengths. This instrument was designed to make such measurements accurately and in such a way that the results could be recorded in numerical values. The instrument was used by Ely and Ross (*Journal of the Franklin Institute*, 242:85, 1946) in the measurement of relative densities of a series of negatives obtained by exposing chicken erythrocytes to various wave lengths of ultraviolet light. This densitometer can also be used for other purposes, such as examination of spectrographic plates.

The densitometer measures currents in a photoelectric cell mounted above one eyepiece of a binocular microscope. The second eyepiece is used for visual observation. Fig. 1 shows the arrangement of light source, microscope, and densitometer circuit case.

Current measurements are made with an electronic current amplifier having a current amplification of about 400,000. Fig. 2 is the circuit diagram. All components are mounted in a brass case which provides complete electrical shielding as well as stray light shielding. The only controls are the power switch S and the potentiometer P. Adjustment of the potentiometer provides the zero setting for the output meter.

The following method is at present used for relative density measurements: A negative is mounted on the microscope stage and illuminated from below by a tungsten lamp in the method used in viewing tissue sections. The film is brought into focus. The circuit case position is then adjusted so that the beam of light from one eyepiece passes through the opening in the bottom of the case and strikes the photoelectric cell. The lamp is then extinguished and the output meter reading adjusted to zero. The lamp is turned on and a portion of the negative containing no cell images is brought into focus under the microscope. The intensity of the light incident on the film is now adjusted by changing the microscope condenser aperture until the output meter reads 10 divisions.

The film position is then shifted until a cell nucleus is in focus under the microscope and the meter reading is recorded. Similar readings are obtained on other cell nuclei on this film.

A new film obtained by exposure of the same cells to a different wave length of ultraviolet light is placed under the microscope objective. With a section of the film containing no cell images in focus, the aperture

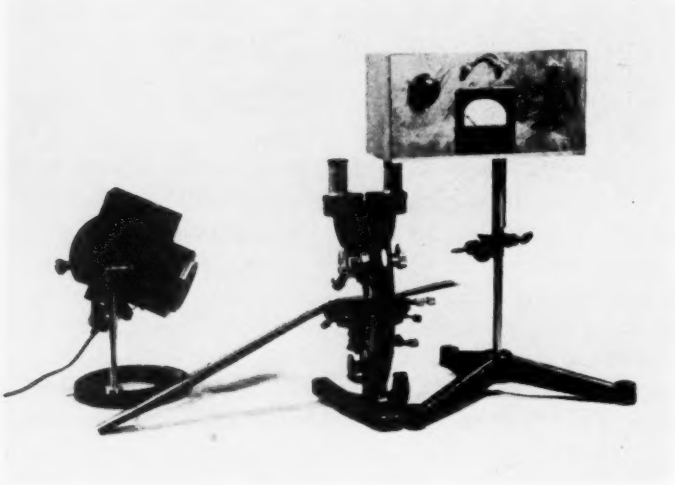

FIG. 1. Densitometer.

is again adjusted to a meter reading of 10 divisions. Meter readings are then taken for a number of cell nuclei.

For each film, the densitometer readings on images of cell nuclei divided by the readings on background areas of film represent the relative light transmission of the photographic images of the cell and of the film. As the background density readings have been equalized for

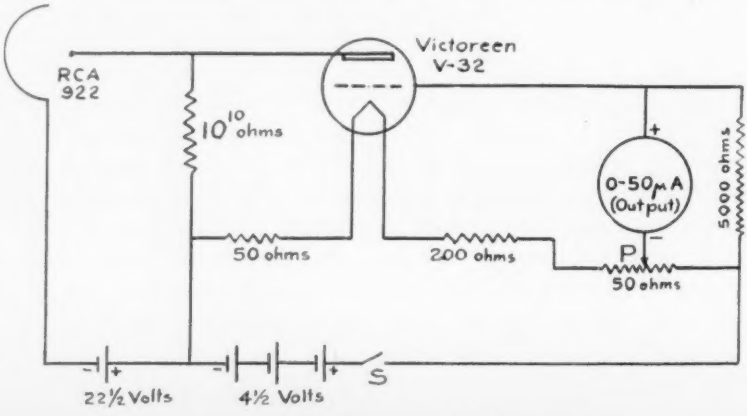

FIG. 2. Circuit diagram of densitometer.

all films, the densitometer readings on the nuclear images are a direct measure of the relative light transmission of the images regardless of variations in film exposure.

Hence if the film density varies inversely as absorption of light by the photographed cells, the densitometer readings vary as the absorption of light by the photographed cells. Thus numerical values are obtained for the absorption of ultraviolet light of different wave lengths by the cells.

BOOK REVIEWS.

THE HEATING OF STEEL, by M. H. Mawhinney. 265 pages, drawings, tables and illustrations, 15 × 23 cms. New York, Reinhold Publishing Corporation, 1945. Price $4.75.

This book brings up to date the progress of the last twenty years in the adoption of modern furnaces for heat treating the finished steel before shipment. In this period metallurgical requirements have become increasingly difficult to meet, and the growth of atmosphere furnaces to produce a clean heated surface and to control the carbon content of the steel during heating has resulted from these metallurgical demands.

The opening chapter deals with the chemical effects of heating steel which includes a discussion of oxidation, carburization, and decarburization of steel when heated under various conditions. This leads into the question of protective gases neutral to carbon as well as to iron. The second chapter reviews the characteristics of the various fuels and describes the various types of burners available for preparing these fuels for combustion. Improved methods for the rapid determination of fuel requirements in furnaces and data on the cost comparison of fuels are included in this chapter. Following this is a discussion on the uniform distribution of heat within the furnace, with the factors which affect this distribution. Mechanical and electrical equipment is described for the control of atmosphere in open furnaces. The calculation of heat transfer by empirical methods have been collected and are discussed as is the distribution of fuels in a typical steel mill.

The quenching of steel after heating is a subject included in the book, giving a brief outline of the theory and a discussion of quenching media and the quantities, circulation, cooling, and control of the various media employed. Also a treatment on alloys and refractories is given. Suspended refractory designs have been worked out to answer certain difficult refractory problems. The last chapter is devoted to giving a bird's eye view of the heating equipment used by the steel industry which is of value in summarizing the problems in the preceding chapters. There is a subject index in the back.

The book gives a worth while modern treatment of those features of heating methods and of furnace tools which are important in obtaining the best results from the heating of steel.

R. H. OPPERMANN.

ATOMIC AND FREE RADICAL REACTIONS, by E. W. R. Steacie. 548 pages, illustrations, 15 × 23 cms. New York, Reinhold Publishing Corporation, 1946. Price $8.00.

Information concerning elementary reactions, which are of major importance in the elucidation of the mechanism of thermal and photochemical reactions, is widely spread throughout the literature of chemical kinetics, photochemistry, pyrolysis, etc. This book brings together such data and treats the reactions of atoms and radicals in their own right rather than as an incidental part of the mechanism of more complex changes. The discussion is confined to elementary reactions involving organic substances.

The first half of the book contains a general discussion of experimental methods and the role of atoms and radicals in thermal and photochemical reactions. The last half treats on systems containing carbon and hydrogen only, oxygen, nitrogen, chlorine, bromine, iodine, sodium, other metals, and sulfur. It is a systematic survey in which an attempt is made to discuss all data which have a bearing on the rates of individual and elementary reactions. In some cases where a great deal of work has been done on a single problem, as for example the acetone photolysis, it is not possible to discuss a single one of the elementary steps involved without detailed reference to the other steps in the mechanism. In such cases the main discussion is given on the mechanism of photolysis and only a brief summary is given in sections dealing with specific elementary steps. In other cases where the material is less voluminous,

the main discussion is given under one of the part reactions involved, and reference is made to this in the discussion of other part reactions.

The book is an American Chemical Society Monograph. The data are the result of an exhaustive search of the literature and it is coupled authoritatively into a work that is a valuable reference tool.

R. H. OPPERMANN.

PETROLEUM PRODUCTION, by Park J. Jones. Volume I. 228 pages, drawings, 15 × 23 cms. New York, Reinhold Publishing Corporation, 1946. Price $4.50.

To designate the portion of coverage of this book, the author has classified the production of oil, condensate and natural gas into mechanics of production; the optimum rate of production; reserves and well spacing; condensate and natural gas production; oil production, primary methods and oil production, secondary methods. This Volume I is limited to the first named.

The elements of production mechanics is first taken up which lays the groundwork for descriptions of characteristics of pay and nonpay media, permeability of linear and radial systems, interstitial and free water and application of electric logs. The composition of hydrocarbons is next taken up including crude oil, molal composition, liquid volume and gas volume composition. This is followed by a treatment on the properties of reservoir gases. Under the heading of separator samples and equilibrium constants, the general method for combining a given rate of liquid production of known composition with the corresponding rate of gas production of known composition is considered. The displacement of oil by water, of oil by gas, of oil and water by gravity are the succeeding subjects. The later part of the book is devoted to invasion factors and patterns, displacement of condensate by cycling, and volumetric balances. There is an author and subject index in the back.

Those interested in the subject will find here a clear presentation valuable as a text and reference.

R. H. OPPERMANN.

SEMI-MICRO QUANTITATIVE ORGANIC ANALYSIS, by R. Belcher and A. L. Godbert. 168 pages, drawings, 14 × 22 cms. New York, Longmans, Green and Co., 1945. Price $3.00.

Semi-Micro methods deal with samples of 20 to 50 mg. Balances are available for the same accuracy of weighing as in the cases of macro and micro methods. The technique, however, is easier to acquire than the micro method and savings of time, space and labor result over the macro or micro methods. This book is devoted to a description of the elementary semi-micro analysis of organic compounds.

At the outset there is an introduction involving briefly a general determination of elements, group estimations, and physico-chemical determinations. The balance and methods of weighing are described as are general apparatus and filtration. Following this, the book is divided into three parts. The first deals in detail with determination of the elements of moisture, ash, metals, carbon, hydrogen, nitrogen, sulphur, halogens, phosphorus, and arsenic. Under groups in the second part are determination of the carboxyl, methoxyl, and acetyl groups. And the final part is concerned with determination of densities of liquids, melting points and boiling points, and molecular weights. The two appendixes are on the purification of small amounts of material, and the preparation and standardization of volumetric solutions.

The book should appeal to the analyst as an explanation of the simplest of the accurate methods.

R. H. OPPERMANN.

TWO-WAY RADIO, by Samuel Freedman. 506 pages, drawings and illustrations, 16 × 24 cms. Chicago and New York, Ziff-Davis Publishing Company, 1946. Price $5.00.

This field has been expanding rapidly as it emerges from war-time applications to peace-time applications. It is currently opening up a billion dollar field of business for the radio industry as well as over one hundred thousand new jobs during the next five years in the United

States. The new frequency spectrum legalized by the Federal Communications Commission has increased by over three hundred times directly and by thousands of times indirectly the channel space available for two way radio communication.

This book describes the mechanics and applications of two-way radio for all forms of fixed, mobile, or portable communications. It is presented in a non-mathematical form and in simple language fully understandable to persons using or intending to use such facilities. Starting with an introduction as to what two-way radio means, it then covers the planning and power supply of such equipment, followed by a treatment of some characteristics of mobile and fixed stations. The more technical aspects are then taken up beginning with the antenna and proceeding through amplitude and frequency modulation equipment and induction radio and carrier systems. A separate chapter is devoted to microwaves which includes frequencies up to 300 megacycles. The author states that this subject can be expanded further particularly in view of the fact that much of the new frequency allocations by the Federal Communications Commission lies above that figure.

The next section of the book is devoted to applications including railroads, police, fire, forrestry, highway, public transportation, marine, aeronautical, and personalized applications such as for doctors, hospitals and ambulances, amateurs, farmers, etc. An important chapter is on maintenance and repair wherein an analysis of troubles is made, a list of typical troubles encountered in operation, and explanations in the use of tools and equipment, and adjusting and tuning. The latter chapters of the book are given to instructions and explanations of licenses and regulations, and typical radio systems. There is a valuable subject index in the back.

The book is a modern presentation of the subject for those whose technical training along these lines is rather limited.

R. H. OPPERMANN.

ELECTRONS IN ACTION, by James Stokley. 320 pages, drawings and illustrations, 14 × 21 cms. New York and London, Wittlesey House, McGraw-Hill Book Company, Inc., 1946. Price $3.00.

That the electronic age is with us cannot be disputed. The tremendous impetus given to the field of electronics by the war will be utilized by the peace and even more wonderful will be the uses of electronics. With all this, the need for knowledge by the layman will be manifest. Today, we are on the threshold of these developments and even now there are innumerable questions, the answers to which would bring more satisfaction and contribute to better living. What are electrons? How are they put to work for mankind? What are the principles of radar, the electron microscope, television, x-rays, atomic energy? These and a host of other questions are given solutions in this book on Electrons in Action.

The book starts with a description of the flow of electricity where the familiar query "What is Electricity?" is treated upon. Electrons naturally come out of this discussion, what they are, how to free them from substances, and to put them to work for mankind. The historical development of radio, the discussion of which includes a simple understandable explanation of operation is given adequate space. A like treatment is given the electrical transmission of photographs, and television. In a chapter on Cosmic Electronics an interesting account is given of studies which may be a powerful new means of learning additional facts about the universe in which we are living.

The book continues with descriptions and explanations of a great many contributions which the theory and applications of electronics have provided. Such descriptions as those given on the fluorescent lamp, electronic heating of metals and non-metals in industry the spectrophotometer, and the use of electronics in medicine are especially good. A final chapter is devoted to atomic energy. It gives a fascinating running account of the development of the nuclear chain reaction leading to the atomic bomb.

Written in a simple style easily understood by the layman, this book will provide the reader a means of greater appreciation of present day and future achievements on electronics.

R. H. OPPERMANN.

ENCYCLOPEDIA OF CHEMICAL REACTIONS, by C. A. Jacobson. Volume I. 804 pages, 15 × 24 cms. New York, Reinhold Publishing Corporation, 1946.

This is the first volume of a set which eventually is to contain all or practically all the published chemical reactions, described briefly and expressed in equation form. It is the beginning of a great work, to continue and be published later as supplements, finally bringing the work up to date and kept so. The system is alphabetically arranged first as to the formulas of the reactants and next as to reagents. This volume covers the chemical reactions under the elements aluminum, antimony, arsenic, barium, beryllium, bismuth, boron, and bromine. The arrangement in detail includes conveniently locating the reacting substances and reagents, a brief statement of the conditions governing the reaction, the reaction in equation form, and the compound precipitated in the reaction.

For convenience in locating any desired reaction, two sets of indexes have been placed in the back of the book. The first lists all reagents used as reacting with a given reactant. The second index lists also alphabetically all the compounds that have been formed or produced in these reactions, giving reference by number to the reaction where the compound in question may be found.

This encyclopedia will fill a great need in facilitating library research, which is basic to chemical progress.

R. H. OPPERMANN.

PHYSICAL CONSTANTS OF HYDROCARBONS, by Gustav Egloff. Volume III. 661 pages, illustrations, 15 × 23 cms. New York, Reinhold Publishing Corporation, 1946. Price $15.00.

This American Chemical Society Monograph is the third of a series containing a collation and evaluation of physical constants of mononuclear aromatic hydrocarbons. This classification excludes hydrocarbons having a fused ring structure, any ring of which is aromatic. Biphenyl and bicycloheptyl-benzine appear in this volume, but polynuclear aromatics such as indane and tetralin will appear in Volume IV. The constants of aliphatic and alicyclic hydrocarbons were published in Volumes I and II respectively.

This is the result of a research which involved a total of about 20,000 references of which approximately 5,500 contained data suitable for this volume and Volume IV. The spread covers the years up to May 1944 from *Chemical Abstracts* and *Chemisches Zentralblatt* through 1939 and the issues available since 1940. Melting and boiling points, densities, refractive indices of the sodium D, hydrogen α, β, and γ, and helium lines, angles of rotation at the sodium D lines, critical temperatures, pressures and densities, and sublimation points were recorded.

The book stands along with its two previous volumes as a standard reference.

R. H. OPPERMANN.

PUBLICATIONS RECEIVED.

David Rittenhouse, by Edward Ford. 226 pages, 14 × 21 cms. Philadelphia, University of Pennsylvania Press, 1946. Price $2.50.

Soul of Lodestone, by Alfred Still. 233 pages, 14 × 21 cms. New York, Murray Hill Books, Inc., 1946. Price $2.50.

Precis de Radioelectricite, by Edmond Divoire. 222 pages, illustrations, 16 × 25 cms. Paris, Masson & Cie, Editeurs, 1945.

CURRENT TOPICS.

Wartime Rubber Statistics.—Full scope of America's victory in its wartime battle for an independent supply of crude rubber was disclosed recently with the release of data on 1945 rubber consumption which emphasizes the staggering output of the nation's 700 million dollar synthetic rubber plant.

The factual tabulation of consumption in all classes of crude and reclaimed rubber covered the years 1941 through 1945, the fifth year of the synthetic rubber program. These figures show how military necessity had driven up production of vital synthetic rubbers from a scant 6,000 tons in 1941 to a record-breaking 798,798 tons last year.

What cold statistics fail to trace, the Rubber Manufacturers Association noted, is the dramatic sequence of events in a nation's life-and-death race for a basic raw material during four critical war years—a race that stands high in industrial history.

The United States consumed 781,259 tons of rubber in 1941. To this total, laboring synthetic pilot plants contributed less than one per cent. of the total tonnage in 1941. Production was still actually on a pilot plant basis when the Japanese swept down through the Malayan peninsula and into the Netherlands Indies, cutting off more than 90 per cent. of the world's natural rubber supply.

Fall of Singapore in early 1942 was the starter's shot that catapulted the nation, its armed forces and three of its major industries—rubber, oil, and chemistry—into a desperate race for new sources of rubber, even for survival.

Success of the government-industry team in attaining mass production of high grade synthetic rubbers before the precious stockpile of natural rubber had melted away is reflected in the table on rubber consumption through the war years.

Chief of the war-winning types of rubber in the production picture was the government's general purpose synthetic, GR-S. In the face of a mad scramble for construction priorities, scarce materials, hard-to-get chemicals, and at times almost non-existent manpower, GR-S consumption jumped from 108 tons in 1941 to 2,579 in 1942; 131,977 in 1943; 495,552 in 1944 and to 600,001 tons in 1945—more than this nation had ever used of all types of rubber in a normal peacetime year.

During that time the outcome of the race with the stockpile was frequently close, for over a period of many months the natural rubber stockpile stood at less than 100,000 tons, the critical danger point defined by the Baruch report early in the war. Natural rubber consumption had shrunk from 775,000 tons in 1941 to 376,791 tons in 1942; 317,634 tons in 1943; 144,113 tons in 1944 and only 105,406 tons in 1945.

Output of other, special purpose synthetic rubbers matched soaring production in the GR-S plants. And with the problem of price under the full focus of industrial research and production know-how, cost factors declined almost as sensationally as production climbed.

Prices established at the opening of the program prevailed until April 1943: GR-S at 50 cents a pound, GR-M (Neoprene) at 65 cents a pound, and GR-I

(Butyl) at 33 cents a pound. At the end of 1945, GR-S was selling at 18½ cents a pound, GR-M at 27½ cents a pound and GR-I at 15½ cents a pound.

Consumption of Crude and Reclaimed Rubber
(In Long Tons).

Year	Natural	GR-S	GR-M	GR-I	All Other	Reclaim
1941	775,000	108	4,463	—	1,688	251,231
1942	376,791	2,579	6,833	22	8,217	254,820
1943	317,634	131,977	26,205	304	12,405	291,820
1944	144,113	495,552	46,243	10,763	14,112	251,083
1945	105,406	600,001	42,363	43,009	8,019	240,780

R. H. O.

Plastic Improves in Heat Test.—After one week's baking over a hot oven, General Electric's new chemical recipe, silicone glass laminate, keeps getting better and better.

The new plastic has far surpassed any similar product by withstanding the week-long test at 250 degrees. More unusual is that both the mechanical and electrical properties of the laminate grow better each day. Pointing out that any other plastic laminate would deteriorate under such conditions long before the new member of the silicone family, G.-E. chemists have plans for still more severe heat tests.

Constructed of layers of glass cloth treated with a silicone resin and subjected to heat and pressure, the laminate is so resistant to heat that whole new areas in the electrical insulation field may be opened. According to Dr. J. J. Pyle, director of the laboratory, ordinary solder will melt before this new insulating material will break down under heat.

Many uses for silicone glass laminates are being found in such electrical equipment as motors, circuit breakers, induction heaters, high frequency oscillators and other electrical and electronic equipment. Other uses may include handles for arc welding electrodes and for indestructible insulating parts for devices subject to fire hazards.

Although heavy for plastics, this material weighs about the same as magnesium.

R. H. O.

Oak Ridge Realized Exceptional Safety Record (*Electrical Engineering*, Vol. 65, No. 5).—Management of the town of Oak Ridge, Tenn., which grew during the completion of the atomic bomb project from an initial population of 500 to one of 75,000, was accomplished by the Turner Construction Company with resulting accident frequency and severity rates lower than the national average. The company, which worked through its wholly owned subsidiary, the Roane-Anderson Company, was responsible for safety measures in both the residential and production areas of the community, as well as for providing every community need of an American city of like size.

Though excess speed normally is the enemy of safety, at Oak Ridge both speed and safety had priorities, with secrecy an ever-present condition for all operations. None of the 12,000 employees of the company knew what was

being made in the production areas. Even Raymond T. Bartnett, chief safety engineer, was ignorant of what was being manufactured.

Utilities, building maintenance, and railroads were assigned to one safety engineer; dormitories, housing and cafeterias to another; medical, cleaning, construction, fuel distribution and refuse to another; salvage yards, administrative, fire department, roads and streets and farm departments to another; supplies, warehouses, coal storage, central warehousing, equipment repair and motor pool departments to another. A fleet engineer was responsible for a vehicle safety program.

Expenditures of the safety department, however, were less than one half of one per cent. of the total company payroll. The entire departmental program was carried on with a total of eight safety engineers, one fleet engineer with a driver training assistant, one statistical engineer, one statistical assistant, and five clerical assistants. With its major responsibility an industrial safety program, the department dealt with every conceivable type of working exposure.

"While we had our share of the more interesting and dramatic problems, such as fumigation with hydrocyanic acid gas and methyl bromide, railway freight operations of labeled cargoes, high voltage operations and maintenance, operation of an abbatoir and others, we still had the usual floor falls and material handling troubles," according to Mr. Bartnett.

In spite of all abnormal circumstances, the above-average safety record was established. The national frequency average computed and weighed for similar operations was 20.27. The Roane-Anderson 1944 cumulative frequency was less than 50 per cent. of this, and for the first eight months of 1945 was less than 30 per cent. of the national average. Severity rates were under 40 per cent. of the national average.

R. H. O.

Metal Lens Developed for Microwave Transmission (*Electrical Engineering*, Vol. 65, No. 5).—A metal lens capable of focusing radio waves as an optical lens focuses light has been developed at the Bell Telephone Laboratories, New York, N. Y. The new lens is expected to find its most widespread application in microwave radio relay systems such as the New York to Boston channel now under construction. These systems similar to systems developed for the Armed Forces during the war, are designed primarily as adjuncts to the telephone network but may find additional use in transmitting pictures, radio-broadcasts, and television programs, as well as in the peacetime development of radar.

A major problem in the development of microwave communication has been that of transmitting and receiving the wave energy in the form of a narrow beam like that of a searchlight. These very short waves do not follow the curvature of the earth but shoot off into space and so transmission over long distances requires the use of relays. The new metal lens produces the sharp beam necessary to obtain short range transmission between successive relay stations with the least interference.

The lens operates on a principle comparable to that of a simple convex magnifying glass which can focus the sun's rays to burn a hole in a sheet of paper. The utilization of this principle is possible because radio waves are of

the same electromagnetic nature as light waves. To overcome the necessity for the huge lens called for by the greater length of radio waves, a system of metal plates is employed which duplicates the action not only of convex and concave lens but also of other optical devices such as half and quarter wave plates and prisms.

Design and development of the metal lens was under the direction of Doctor Winston E. Kock who in 1938, was chosen by Eta Kappa Nu as the outstanding young engineer of the year. Doctor Kock was the third man to be so honored.

R. H. O.

Hydroelectric Developments in Various Countries(*Electrical Engineering*, Vol. 65, No. 5).—With the development of the Paulo Affonso Falls of the São Francisco River, Brazil, along the lines of the Tennessee Valley Authority, a potential hydroelectric energy estimated at 608,000 horsepower or 448,000 kw. will be made available to northeastern industry of Brazil and water and power will be provided for the irrigation of lands which otherwise would be useless. As a result this region, which now is dependent upon southern Brazilian states for agricultural and manufactured products, will be able to supply many of its own needs as well as those of other parts of the country.

In Eire two dams and two power stations will be constructed on the River Erne between Belleek and Ballyshannon. The power stations will be located near Cliffhouse and Cathaleen's Falls and at first will produce an anticipated annual output of 200,000,000 kilowatt-hours which, with full development, will rise to 250,000,000 kilowatt-hours. It is estimated that three years and 1,000 workers will be required to complete the project.

Portugal currently has a total of 110 hydroelectric power plants, operating on 11 rivers, with a total capacity of 99,009 kw. generating 204,971,101 kilowatt-hours of electricity. If present plans for dam and plant construction materialize, current in Portugal will be entirely hydroelectric, and output is expected to increase from the 470,000,000 kilowatt-hours used now to 1,500,000,000 kilowatt-hours in 1952. Consumption of electricity in 1943 totalled 397,452,345 kilowatt-hours of which 253,366,758 kilowatt-hours were consumed by Portuguese industry. Annual household consumption in Portugal is low; about 60 kilowatt-hours per capita as compared with 150 in Spain, 300 to 500 in Switzerland, 1,000 in Norway, and 1,225 in the United States.

Plans for the construction of an underground hydroelectric station on Lake Sevan, Armenia, and of 973 smaller hydroelectric stations throughout Russia recently were reported in the Soviet press.

The Lake Sevan project will entail construction of a 5½-kilometer tunnel through which the water from the lake will be directed. The high altitude and shape of the lake bottom at present cause evaporation of up to 97 per cent. of its water. In addition to providing irrigation to a large farming area, the development is expected to influence the growth of chemical, rubber, aluminum, and food industries in Armenia.

The proposed increase in the number of hydroelectric stations is a notable one when it is remembered that the total number of such stations was 800 at the beginning of 1945. Also reported under construction were 276 small steam power stations. This activity is said to be particularly widespread in

the Urals. Of the 103 units being set up in October 1945 by collective farmers in the Molotov region, 43 were already in service. In the Sverdlovsk region of the Urals 179 small hydroelectric stations were under construction in the fall of 1945, as a step toward the goal of the farmers of the region to bring electricity to 1,000 villages in 1945. Electrification of the remaining rural localities in the region has been planned for 1946.

Specifications for a hydroelectric project at Aswan, Egypt, which will require three years for completion and which will develop 500,000 horsepower have been prepared in London, the Egyptian press reports. A unique feature of the project is that the removal of the surface earth or underlying granite in the vicinity of the dam must be done by mechanical means rather than by dynamiting, to avoid the danger of weakening the twice-heightened dam.

An agreement has been reached for the preparation of plans for hydraulic works in the mountains near Athens, Greece. Preliminary studies and scientific research have been made toward the creation of artificial lakes to furnish 1,830,000,000 cubic meters to the Athens water supply. This is expected to provide electric energy from water power amounting to about 687,000,000 kilowatt-hours a year.

R. H. O.

Greatest Achievements of Atomic Science Will Be Ones Now Unforeseen.—The most important future applications of nuclear, or "atomic," energy will be the ones that cannot now be foretold, declared Dr. Laurence A. Hawkins, consultant of the General Electric Research Laboratory, who spoke in York, Pa., before the University Club.

He compared the present state of physical science to that of Europe just after Columbus announced the discovery of a new world. Though hopes of an Eldorado, a fountain of youth and a northwest passage to India were not fulfilled, the New World actually yielded unanticipated riches far greater than any of these.

"So now," he stated, "as we face the new world within the atom's nucleus, which has been opened for our exploration, we may feel sure that however baseless our present speculations may prove to be, we shall attain through further resourceful research to riches now unforeseen."

Dr. Hawkins called attention to the difference between fundamental and applied research. The former seeks for new knowledge for its own sake, usually with no practical objective in view, while applied research is aimed at new knowledge about a specific thing.

"War research," he said, "was applied research. Even the work on the atomic bomb was applied research and development for all the basic knowledge concerning chain reactions with U-235 was pre-war. During the war, fundamental research was at a standstill. There was no time to explore new fields in the hope of finding something useful; the need was to apply existing knowledge to military problems with the utmost effectiveness and speed.

"It was a race in technological development in which the Germans had a long head start. How narrow the margin of victory was, no one fully realized until the Allies overran Germany and discovered the near completion of facilities for quantity production of robot bombs which, in another six months, would have blasted most English cities and air bases off the map."

Though as regards its future commercial applications, it is always a gamble, "fundamental research is most important of all," continued Dr. Hawkins. "We cannot have too much of it, and it is to the universities that we must look for most of it. University funds for research are now inadequate, so that government aid is much needed, and, if granted with no strings attached beyond the requirement of full publication of results, it should prove a most profitable investment."

He told the audience that the present bill pending in Congress for support of research, representing a compromise between the original Kilgore Bill, to which the scientists strenouusly objected, and the Magnusson Bill which they endorsed, seems workable and is therefore acceptable. He also endorsed the MacMahon Bill for the control of atomic energy, which is also a compromise. "This bill," he said, "satisfies all but the military extremists, and is another demonstration of the ultimate success of democracy in action which has come to comfort us in the welter of confusion and blundering which has brought us so close to disaster at home and abroad."

Dr. Hawkins deplored early official utterances in regard to our new knowledge of atomic processes, "which seemed to declare our intention of maintaining secrecy as fully and as long as possible. Nothing could have been better calculated to poision the international atmosphere, to weaken the United Nations, and to precipitate a new armament race toward another and most horribly destructive war. It is to be fervently hoped that from now on it will be made clear that we are firmly committed to the wise program developed by the so-called Lillienthal Committee."

R. H. O.

"Shooting Star" Engine is Triumph for Modern Welding.—Utilization of many specialized types of welding to produce joints of great strength in lightweight metal was one of the main factors in the successful development of the General Electric Type I-40 jet propulsion gas turbine, which powered the Lockheed Shooting Star on its recent record-breaking transcontinental flight.

The plane flew from Long Beach, California, to La Guardia Field, New York, in four hours, 13 minutes, and 26 seconds, averaging 584 miles an hour on the trip and at times reaching speeds in excess of 600 miles per hour.

One of the chief features in the all-welded construction engine, besides the lightweight metals used, is the extremely favorable horsepower-weight ratio of better than two horsepower per pound, as compared with one per pound in reciprocating type engines.

There are more than 500 welded joints in the engine, and in making them practically every known type of welding was utilized, each one in applications where its particular advantages could best be employed. For example: the circular seam welding of a flange to the outer exhaust cone is done by resistance seam welding, and many parts in the exhause unit assembly itself, where pressure tight welds are not required, are spot welded.

Since the intense heat of the combustion creates very high operating temperatures (about 1,500° F.), heat-resistant alloys are required for the combustion chamber and the exhaust unit. In order to keep the weight of the engine down, these heat resistant alloys were used in sheet form, some as light as 0.022 inch in thickness.

These austenitic stainless alloys give high strength and good corrosion

resistance at high temperature. By welding these parts the engineer was able to design for minimum weight with maximum joint efficiency, both in thermal strength and physical strength.

Fabrication of various parts out of sheet metal, however, led to one of the real production problems and it was only through the closest co-operation between the design engineers, manufacturing department and the welding engineer that the correct procedure for making parts was finally established.

The fabrication included joint design, fixturing, welding process and machining procedure. In considering joint design, it was first necessary to design for strength. Then weight, distortion, and gas flow were closely checked. For example, a joint where it was necessary to join pieces of different thickness, it was known that minimum weight would be obtained by using a straight butt joint. A joint of this type, however, would set up a stress concentration in the thinner piece and encourage failure in service. Fusion welding of this joint would also cause greater distortion. Therefore, where the flanges were welded to the exhaust casing or the ring holders as well as to the flame tubes, a lap joint was used and the weld was generally made by resistance seam welding. This joint proved tight, had excellent strength, a minimum of distortion, and a neat appearance.

In cases where there was great variation in thickness, it was possible that fatigue failure would occur in the thinner section, next to the lap. Because of this, it was desirable to keep both pieces as nearly alike in thickness as possible. The longitudinal seams, in some instances, were designed for a flush joint without the necessity of grinding. These were made by clamping the joint in a fixture and backing it with hydrogen, and then welding it by the atomic-hydrogen process. The hydrogen backing was provided by burning hydrogen in a groove milled in the backing bar of the fixture. On the thicknesses up to $\frac{1}{16}$ inch, it proved to be possible to butt the edges up square, and, using the fixture as mentioned, weld the seam without the addition of filler or the use of flux. This resulted in a nearly flush joint that did not require cleaning and had excellent physical strength. The use of filler was eliminated by clamping both sides of the joint tightly within $\frac{1}{4}$ inch of the joint. Bringing the heat of the arc to the metal expanded it and forced it to hump at the joints. This hump was melted down, giving a flush weld. As the metal cooled, a little elongation occurred between the clamps, but this was so slight that it had little effect upon the strength of the joint. In some cases where a fillet weld was necessary between sheet stock and a heavier section, the metal arc welding process was used to good advantage.

It is axiomatic that distortion caused by welding, especially in sheet metal parts, is practically unavoidable. In order to make use of the advantages of these parts, however, allowance had to be made for this distortion wherever possible. When this allowance could not be made, however, it was possible to reduce the distortion by using proper fixtures and sequence. In some cases, welding sequence can be used to reduce distortion and machining sequence to correct it.

By careful planning of operations in manufacturing the Type I-40 engine, it was possible to solve these problems and to make parts according to the manufacturing drawing, in spite of the many problems encountered in welding on sheet metal.

R. H. O.

New Improvement Will Raise Energy of Betatron from 100,000,000 to 160,000,000 Volts.—By a relatively simple improvement, the 100,000,000-volt betatron in the General Electric Research Laboratory, most potent atom-smasher now in operation, may soon have its output energy raised to 160,000,000 volts. This was announced by Dr. C. G. Suits, G-E Vice President and director of the Research Laboratory, who credited W. F. Westendorp, of the Laboratory's X-Ray Section, with responsibility for the improvement.

The betatron operates on principles similar to that of a transformer, in which the flow of alternating current back and forth in one coil, called the primary, produces a comparable flow in another coil called the secondary. Primary and secondary are connected magnetically, since they are both wound on the same iron core. The ratio between the voltage in the primary and that in the secondary is the same as that between the turns of wire in the two coils.

Alternating current, as commonly used, reverses itself 120 times each second. Starting from zero, it flows one direction, building up to maximum voltage, then returning to zero. After that it does the same thing in the opposite direction with 60 complete cycles each second. An electric current consists of a flow of electrons between atoms. Thus the electrons in the secondary of a transformer are constantly oscillating back and forth, first drifting in one direction, then, 1/120th second later, going the other way.

In the betatron, the secondary coil is replaced by a vacuum tube called the "doughnutron" from its shape. The electrons originate from a heated filament and flow in open space instead of through the atoms of a wire. They are held in their proper orbit by the magnetic field of the huge electromagnet in which the doughnutron is placed.

If the electrons remained in the tube continually, they would first revolve one way, then stop and reverse, but the trick is to get them out while they are moving at maximum energy. As the current builds up from zero to its maximum voltage in the first quarter cycle, lasting 1/240th second, they make 250,000 turns and travel 800 miles. Then, before they go into reverse and lose energy, they are shifted from their orbit, and made to hit a tungsten target, generating a beam of high-voltage X-rays.

For a full half cycle, from the time the current is at a maximum in the negative direction, while it is returning to zero, and then building up in a positive direction, the voltage is positive and in the proper sense to speed the electrons. This offers the possibility of accelerating them for nearly twice as long as with the present method and consequently increasing their energy. However, the difficulty is that while the voltage is passing from zero to its positive maximum, the magnetic field is in the wrong direction for guiding the electrons in their orbit.

The method Mr. Westendorp devised for overcoming this difficulty is called "DC bias." In addition to the alternating current, a direct current can be applied to the electromagnet. This shifts the zero line from the middle of the current wave to its bottom, and nearly the whole of the rise can be utilized.

"By the use of this principle, and some other improvements that may be used with it, it is possible to get substantially higher energies with the same weight of magnet," said Dr. Suits. "We are modifying our 100,000,000-volt machine for biased operation and expect to obtain 160,000,000 volts with relatively small changes in the core of the magnet.

"Since no serious difficulties of this method have yet been discovered, the

biased betatron may very well replace the earlier type, especially at higher energies. A 10,000,000 volt biased betatron has already been built in our laboratory for experimental operation. One for 50,000,000 volts is now under construction for further study of the principle."

Dr. Suits also revelaed that the G-E laboratory is constructing two other devices for producing high-energy particles for atomic study. One is the "synchrotron," devised independently by a Russian physicist, J. Veksler, and Dr. E. M. McMillan, of the University of California.

"This device," he said, "employs a magnet and a doughnut-shaped vacuum tube similar to the betatron. The vacuum tube, however, incorporates a high energy resonant cavity energized by a radio frequency oscillator.

"Electrons are injected into the doughnut cavity at relatively high energies and take a circular path because of the magnetic field. The chief accelerating force is provided as the electrons cross the resonant cavity, very much as in the cyclotron. That is, before the negatively charged electrons reach this cavity its polarity is changed so as to give them an additional push. By the use of this principle one may hope to accelerate particles to still higher energies than with the biased betatron, using a given weight of magnetic material."

Dr. Suits said that the G-E laboratory has under construction a modified synchrotron, or "sigmatron," as it is also called, for 70,000-000,volt particles. Following a design of Dr. Herbert C. Pollock, of the laboratory's nuclear physics group, it will be combined with the betatron principle, which will speed electrons to 2,000,000 volts energy before they are fed into the synchrotron cavity. This principle is also being used on other large synchrotrons now under construction.

The other new atom-smashing device which G-E scientists are buidilng is an outgrowth of wartime work in radar. This greatly advanced the art of radio circuitry and electronics at very high frequencies," explained Dr. Suits. "From this work has come a very much fuller understanding of the conduction of electric waves through hollow guides, as well as the development of tubes for the production of microwave radio energy at high power levels. These two advances provide the basic elements of a linear accelerator.

"In one form this device may employ a hollow tube energized by a very high frequency oscillator. Since even the very high power levels available are insufficient for continuous operation, the radio frequency power is applied in regular pulses. Electrons injected into one end of this wave guide may ride the travelling electric wave from one end of the tube to the other, and be thereby accelerated to high energies.

"To adjust the electric wave velocity in the guide to the increasing velocity of the particle during acceleration, the accelerating tube is generally built in a series of tuned cavities. The electric constants of these cavities permit a choice of wave velocity, so that either electrons or heavier particles may be injected and successfully accelerated on their straight path down the axis of the wave guide."

The G-E linear accelerator, one of a number being constructed in various laboratories in the country, is an experimental one to permit an evaluation of the advantages or disadvantages of the method. With a wave guide length of about 5 feet, it will accelerate electrons to 4,500,000 volts. It is being built under the direction of E. J. Lawton and W. C. Hahn.

R. H. O.